Chem 542

Homework for Part 8: Quantum field theory

1. In class, we mostly considered eigenstates of the number operator n = a^+a as our basis set for the radiation field. Such a light beam would have perfect constant intensity. In contrast, eigenstates of the lowering operator a have eigenvalues $\alpha = \omega < q > +i < p >$, from the usual definition of a in terms of q and p (or E and B). Such states would therefore have a well-defined phase $\varphi = \tan^{-1}(q/p)$. This is the case for phase-coherent laser beams.

a. What are the intensity statistics of a laser beam? To answer this, show that the eigenstates of a are given by

$$|\alpha> \sim \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n>.$$

(Hint: insert a complete set of |n> states into $|\alpha> = |\alpha>$, then apply the lowering operator to derive a recursion relation $c_{n+1}\sqrt{n+1} = \alpha c_n$ for the series coefficients $c_n(\alpha) = <\alpha|n>$.)

b. How does the probability of measuring n photons in state |α> depend on n?

Your answer is known as "Poisson statistics," and laser intensities indeed fluctuate according to these statistics. Intensity and phase are conjugate variables, and cannot simultaneously well defined. It is also possible to create "squeezed" photon states with characteristics between |n> and |α>. See "Saleh & Teich" or "Yariv" or a similar quantum electronics text for details.

2. Show that upon integration over the polar angles θ and φ, and summation over the two orthogonal polarizations, the total spontaneous emission rate becomes

$$\Gamma^{tot} = \frac{\omega^3}{2\pi\hbar c^3} \sum_{\lambda} \int d\Omega \left|\vec{\mu}_{fi} \bullet \hat{e}_{k\lambda}\right|^2 = \frac{8\pi}{3} \frac{\omega^3}{2\pi\hbar c^3} \left|\vec{\mu}_{fi}\right|^2$$

(Hints: $d\Omega = \sin\theta \, d\theta \, d\varphi$; you'll have to use the fact that the two $\hat{e}_{k\lambda}$ are mutually ortogonal, and also orthogonal to $\hat{k}$, a unit vector pointing along **k**; this allows you to replace a sum over $\hat{e}_{k\lambda}$ by a single term containing expressions such as $\mu_{fi} \cdot \hat{k}$).