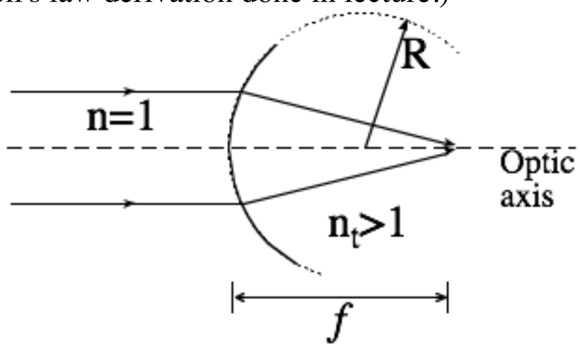
Chem 542

Homework for Part 6: n-level systems, optics and lasers

1. Use Fermat's principle to show that rays of light traveling parallel near the optical axis at n=1 into a glass surface with radius R and n_t>1 will focus. What is the focal length in terms of R and n_t? (Hint: do it in analogy to the Snell's law derivation done in lecture.)



2a. Consider a 1 cm tall "arrow" object 8 cm from a 5 cm focal length thin lens. Use two rays to trace the image of the arrow object. What is the image distance? The magnification?

b. Combine this 5 cm lens with a 100 cm paraboloid mirror in a Newtonian configuration (one focal plane of the lens and the focal plane of the mirror coincide, to form an image at the other focal plane of the lens). Show by calculation or by a drawing that the angular magnification of this system for far-away objects is 20.

3. Use the gaussian E(z,r) from lecture to show that it satisfies the wave equation.

4. Show that for the four level system discussed in class, the population inversion indeed saturates at high intensity (or u), i.e.

$$n_2 - n_1 = \frac{\Delta n_0}{1 + T_{21} B u}$$

5. A He-Ne laser cavity has length 50 cm. How many modes ν_m can lase within the 1.5 GHz gain bandwidth of the He-Ne gas mixture? [Hint: what do you think is the refractive index of this dilute gas?]

6. The gain profile of an Argon ion laser has a spectral width (fwhm) of 6 GHz. What is the shortest mode locked pulse you can obtain from this laser?

7. A Ti:sapphire crystal has an intensity-dependent refractive index $n(I)=1.76+n_2I$, with $n_2=3\cdot10^{-16}$ cm^2/W. Consider a low power pulse (100 W) vs. a femtosecond pulse with $I(t=0)=10^{14}$ W/cm^2 propagating through a 1 cm long Ti:sapphire rod, both collimated Gaussian with $w_0=0.1$ mm >> $\lambda=800$ nm as an approximation. Both pulses cause a Gaussian refractive index gradient in the rod. Note that according to Fermat's principle, a higher refractive index n(I) in the center of the rod is equivalent to a greater optical path at the original refractive index of 1.76, i.e. the rod is equivalent to a Gaussian lens shape! Estimate the focal length of the low power vs. the fs beam for this Kerr 'lens' near the optical axis. [Hint: a Gaussian's second derivative at R=0 is equivalent to a radius of curvature of a spherical lens, such as in problem 1.]