Homework H8 Solution

1. Turn in Problem 1.8 (Page 25)

Solution:

Writing the wavefunction as $\Psi(x) = e^{-|x|}$ and integrating the square modulus over all space we have

$$P = \int_{-\infty}^{\infty} \psi^* \psi dx = \int_{-\infty}^{\infty} e^{-2|x|} dx = 2\int_{0}^{\infty} e^{-2x} dx = -e^{-2x}|_0^{\infty} = 1 \qquad (1)$$

Thus the wavefunction is already normalized! NOTE: We have taken advantage of the symmetry of the function about the y-axis to re-write the integral with new limits of integration in (1).

To calculate the probability of finding the particle between -1 and 1, we simply use the results we obtained in equation (1) but with different limits of integration:

$$P = \int_{-1}^{1} \psi^* \psi dx = \int_{-1}^{1} e^{-2|x|} dx = 2\int_{0}^{1} e^{-2x} dx = -e^{-2x}|_0^{1} = (1 - e^{-2}) = 0.865 \qquad (2)$$

Note: although $|\Psi|^2$ is largest at x=0, the probability of finding the particle exactly at x=0 is zero! After all, $\int_0^0 \psi^* \psi dx = 0$ for any function. That's why $|\Psi|^2$ is the probability per unit distance, and if the distance range is zero, then P=0.

2. Problem 1.9 (Page 25)

Solution:

Section 1.4.5 in the book has some criterion for acceptable wavefunctions. Reading this short section is important to doing the problem. Looking at each wavefunction individually, we observe the following:

i) This wavefunction tends to infinity as $x \rightarrow \pm \infty$ which means that it cannot be normalized and is therefore not acceptable.
ii) This wavefunction is satisfactory.
iii) This wavefunction is not defined at x=3 (it is infinite at this point) and is therefore not acceptable because it is not continuous at all x.
iv) This wavefunction tends to infinity as $x \rightarrow -\infty$ which means that it cannot be normalized and is therefore not acceptable. It is noteworthy to observe, however, that if the domain were restricted to $[0,\infty)$, the wavefunction **would**

be satisfactory. One must take care always to check the form of the function and the domain over which it is defined, and not ubiquitously associate one functional form with a satisfactory/unsatisfactory wavefunction.

3. a. Explain in words why <A> = $\Sigma\, a_n\, P(A=a_n)$ is the average value of A, if A can only have values a_0, a_1, ... a_n.

Solution:

Probability is a statement about the likelihood of obtaining a certain outcome. In our case, that outcome is the value of a measurement associated with an operator. So what can we make of the expectation value (average value) of an operator? It is simply a sum of all possible values with each value being weighted by how likely it is to be obtained. This should make intuitive sense, and is consistent with the above formula.

b. Let's check this with an example: Let's say A is the value of various bills (Like $1, $5, $10, $20 etc). You have three $1 bills and one $5 bill, so $a_0=1$, $a_1=5$, $P(A=a_0=1) = 3/4$, $P(A=a_1=5) = 1/4$. What is <A>? Does this match your intuition for the average value of one of the four bills?

Solution:

Using the formula supplied in part (a) we obtain

$$\langle A\rangle = a_0\left(\frac{3}{4}\right) + a_1\left(\frac{1}{4}\right) = 1\left(\frac{3}{4}\right) + 5\left(\frac{1}{4}\right) = 2$$

If this seems strange to you at first think again about what we have calculated here. We have defined a system with a set number of outcomes (possible values of measurements in the quantum domain), and then calculated the average. The average comes out closer to 1 than to 5 and we should certainly expect this! We have *three* one-dollar bills and only a single five-dollar bill. Therefore, it should intuitively make sense that the average will come out closer to 1 than to 5.

c. According to postulate (4) what is the probability $P(A=a_n, t)$, of measuring the value of observable A to be equal to a_n at time t?

Solution:

The solution can be observed by looking up postulate (4) on the website. It is rewritten here without bra-ket notation: $P(A=a_n, t) = |\int dx \Psi_n^*(x)\Psi(x,t)|^2$. The more the function $\Psi(x,t)$ looks like the function $\Psi^*_n(x)$, the bigger the overlap integral, and the larger the probability that the value a_n for observable A will be observed. If $\Psi(x,t) = \Psi^*_n(x)$, the the probability will be 1 or 100%, and you are guaranteed to observe the value a_n.