Homework H19 Solution

1. Problem 5.1 in the book. Remember from Lecture 13 that this is indeed one of the two degenerate eigenfunctions for the particle on a ring/circle, which we combined and generalized as $e^{iM\phi}$.

Solution: For normalization, $\int_0^{2\pi} d\phi\, \psi * \psi = 1$

$$N^2 \int_0^{2\pi} d\phi\, sin^2(\alpha\phi) = 1$$

$$N^2 \int_0^{2\pi} d\phi\, \frac{1}{2}(1 - cos2\alpha\phi) = 1$$

$$\frac{N^2}{2}\int_0^{2\pi} d\phi - \frac{N^2}{2}\int_0^{2\pi} d\phi\, cos2\alpha\phi = 1$$

$$N^2\pi - \frac{N^2}{2}\left[\frac{sin2\alpha\phi}{2\alpha}\right]_0^{2\pi} = 1$$

$$N = \frac{1}{\sqrt{\pi}}$$

only if α takes positive integer values, the second term on the left equals zero.
So, $\alpha = 1,2,3,..$

Turn in 2. Problem 5.3 in the book. This is an important exercise as it will help you to think about eigenvalues of angular momentum operators, and the number of different values l and s can take.

Solution: Recall the equations for the magnitude of the angular momentum and its projection on the z-axis:

$$|L| = \hbar\sqrt{l(l+1)} \qquad (1)$$

$$L_z = \hbar m_l \qquad (2)$$

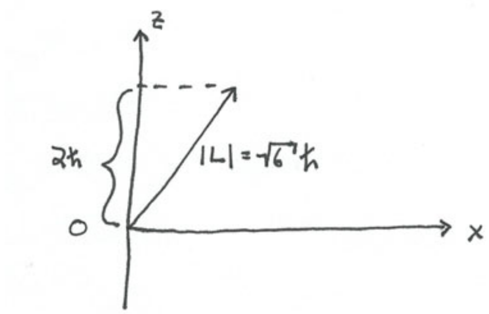
The analogous quantities for spin angular momentum are defined in a similar manner:

$$|S| = \hbar\sqrt{s(s+1)} \qquad (3)$$

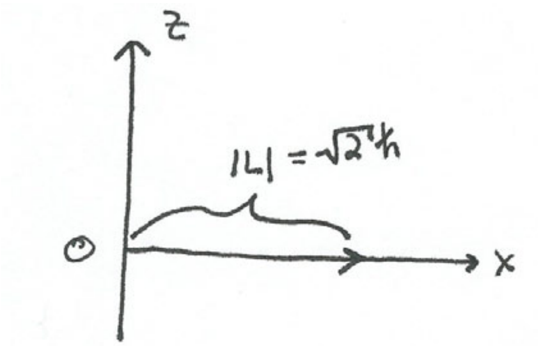
$$S_z = \hbar m_s \qquad (4)$$

In the case of all vector diagrams, these quantities should be calculated and expressed on the diagrams to lend specificity. See next page. Keep in mind that the vector diagrams are classical analogs. You should not interpret them to mean that L does not point quite along the z-axis if $m=l$. Rather, l deviates from the z-axis by a random amount when measured, so that Heisenberg's principle is satisfied because |L| and its conjugate angle φ_L are Fourier-conjugate variables. And note that this is true for ANY axis, not just for the z-axis.

a) $l = 2$, $m_l = 2$



b) $l = 1$, $m_l = 0$



c) $S = 1/2$, $m_s = -1/2$