Homework H17 Solution

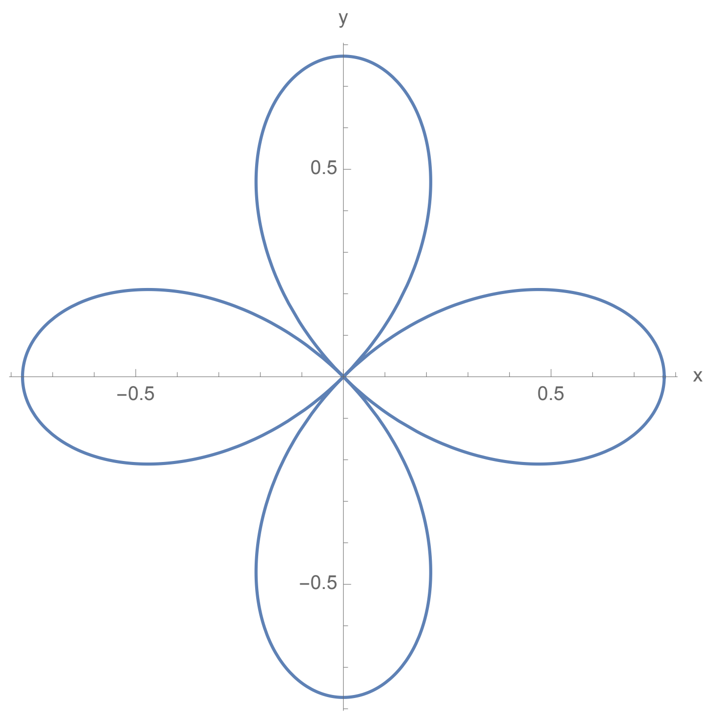
1. Make a plot of the linear combination of the linear combination $Y_{2-2} + Y_{22}$ in the x-y plane ($\theta = \pi/2$).

Solution:

The form of the Spherical Harmonics we need here are:

$$Y_{2,\pm 2} = \frac{1}{4}\sqrt{\frac{15}{2\pi}}e^{\pm 2i\varphi}sin^2(\theta)$$

Setting $\theta = \pi/2$ and making a polar plot in the x-y plane we have



The conventional name for this wavefunction is a d orbital, specifically the $d_{x^2-y^2}$ orbital!

Turn in 2. Problem 6.1 (page 111)

Solution:

To find the energy required to excite an electron from the n=1 to n=2 level in the hydrogen atom, we use

$$\Delta E = E_2 - E_1 = hcR\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \quad (1)$$

$$= (6.626\times10^{-34}\,Js)\left(2.998\times10^{8}\,\frac{m}{s}\right)(1.097\times10^{7}\,m^{-1})\left(\frac{1}{1} - \frac{1}{4}\right) = 1.634\times10^{-18}\,J$$

We must now convert this answer to eV:

$$\Delta E = (1.634\times10^{-18}\,J)\left(\frac{1}{1.602\times10^{-19}\,J\,(eV)^{-1}}\right) = 10.20\,eV$$

3. Problem 6.5 (page 112)

Solution:

To begin, first note the normalized form of the 1s wavefunction

$$\psi_{1s} = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0} \quad (2)$$

This wavefunction only depends on r; as such, when finding the expectation value we may integrate over the ϕ and θ integrals $\int_0^{\pi} \sin(\theta)\,d\theta \int_0^{2\pi} d\phi$ giving a value of 4π, and multiply this constant by the remaining integral:

$$\langle r \rangle = 4\pi \int_0^{\infty} \psi_{1s}^* r \psi_{1s} r^2 dr = \frac{4}{a_0^3}\int_0^{\infty} r^3 e^{-2r/a_0}\,dr$$

$$= \frac{4}{a_0^3}\frac{3!}{\left(\frac{2}{a_0}\right)^4} = \frac{3}{2}a_0 \quad (3)$$

Here we have used the result:

$$\int_0^{\infty} x^n e^{-bx} dx = \frac{n!}{b^{n+1}} \quad (5)$$

to obtain the answer.

4. Problem 6.8 (page 112)

Solution:

To start our detective work, lets find the value of *n* associated with the electron. We know that it takes 3.4 eV of energy to remove an electron from the atom. “Removing” an electron from the atom implies a transition from the current energy level to $n = \infty$. From here we can find out what level the electron currently occupies.

$$\Delta E = E_\infty - E_x = hcR\left(\frac{1}{n_X^2} - \frac{1}{n_\infty^2}\right) = hcR\left(\frac{1}{n_X^2}\right) \tag{6}$$

Where we have simplified (6) using the knowledge that $1/\infty = 0$. Solving for n_X gives

$$n_X = \sqrt{\frac{hcR}{\Delta E}} = \sqrt{\frac{(6.626\times10^{-34}\ Js)\left(2.998\times10^{8}\,\frac{m}{s}\right)(1.097\times10^{7}\ m^{-1})}{(3.4\ eV)(1.602\times10^{-19}\ J\ (eV)^{-1})}} = 2.0002 \tag{7}$$

So $n_X = 2$. We know that that values of n used in equation (6) are *integers,* so any values we get out of equation (7) corresponds to an integer of that magnitude.

Now, we know that the total angular momentum is $\sqrt{2}\hbar$ and we know the formula for the magnitude of total angular momentum:

$$|L| = \hbar\sqrt{l(l+1)} \qquad l = 0,1,2,\ldots \tag{8}$$

It is quite clear from inspection that $l = 1$ leads to a magnitude of $\sqrt{2}\hbar$.

Finally, we are told that no component of the angular momentum lies along the z axis (at least no component beyond what is required by Heisenberg's principle!) This indicates that we are in an M=0 state. So, to summarize we have n=2, l=1, and M=0.

The electron occupies the $2p_z$ orbital.