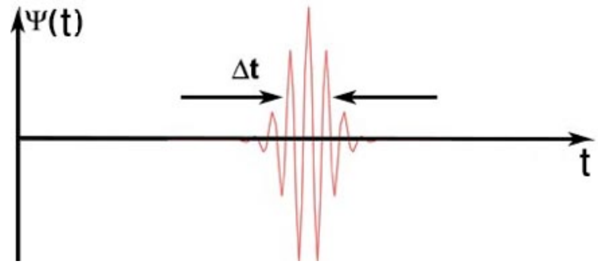
Chem 442: Homework HT2 (2nd TA lecture)

(only turn in **BOLD** assignment; do <u>all</u> assignments by first lecture next week)

Turn in 1. In class, we discussed the Fourier transform of the function $\Psi(x)$=exp[-$(t/2\Delta t)^2$]cos($2\pi t/\tau_0$). It had a peak of width ~$1/\Delta t$ at frequency $\omega_0=2\pi/\tau_0$. This function is like the real part of the wavefunction from homework HT1. A sketch of the function is shown below, roughly indicating the width Δt:



a. Now imagine you make Δt much bigger. Sketch what the function $\Psi(t)$ then looks like. What is the function $\Psi(t)$ when Δt becomes infinite? [Hint: exp[0]=1]

b. Now sketch what the Fourier transform looks like for the original function, and for the new function when Δt gets much bigger. [Hint: remember that $\Delta\omega \sim 1/\Delta t$, as we discussed in class.]

c. Now calculate the Fourier transform of cos($\omega_0 t$) using the integral we defined in class:

$$\Psi(\omega) = \int_{-\infty}^{+\infty} dt\Psi(t)e^{i\omega t} = \int_{-\infty}^{+\infty} dt\Psi(t)\cos\omega t + i\int_{-\infty}^{+\infty} dt\Psi(t)\sin\omega t \ ,$$

giving the average value of $\Psi(\omega)$ at three values of ω: any $\omega \neq \omega_0$, then $\omega=\omega_0$, and $\omega=-\omega_0$. First sketch the integrands $\Psi(t)\cos\omega t$ and $\Psi(t)\sin\omega t$, it will allow you to see when an integral averages out to zero. [Hint: don't be surprised if you get zeros or infinities as answers!]

This kind of function $\Psi(\omega)$, which is like a super-sharp spike at only one frequency (either $+\omega_0$ or $-\omega_0$) is called a "delta function." It just tells you that cos($\omega_0 t$) has only <u>one</u> frequency (ω_0) in it, so the spectrum $\Psi(\omega)$ is a spike at just one frequency. That's what we call "perfect pitch" !

2. Work out the Fourier Transform of Y(t)=sin($\omega_0 t$) in a similar way. Now the big question: if you wanted to know the Fourier transform of sin($\omega_0 t$) +cos($\omega_0 t$), do you have to do any more work? Can you write the Fourier Transform of sin($\omega_0 t$)+cos($\omega_0 t$) in terms of the Fourier transforms of sin($\omega_0 t$) and cos($\omega_0 t$)? This property is called "linearity." You are familiar with it from algebra: if y=x^2 and z=x^3, then the new function f=y+z is just x^2+x^3, so <u>addition</u> of two functions is a linear operation. Is squaring a linear operation?