stationary states = eigenstates

Reviewing Postulate 4:

$$P(A=a_n)=\left|\int_{-\infty}^{\infty} dx\, \psi^*(x)\, \psi(x,t)\right|^2$$

overlap of state of the particle ~~function~~ and an eigenstate

$$\hat{A}\psi_n = a_n \psi_n$$

operator   eigenfunction

Example: $A = x \Rightarrow P(x=x_n) = \underbrace{|\psi(x_n,t)|^2}$

probability ~~for~~ you would find particle @ x_n

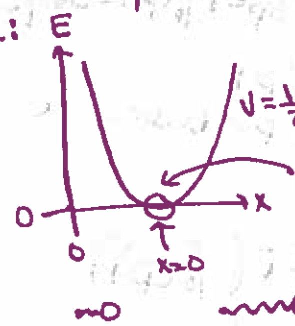
Also provable:

$$\underbrace{\bar{A} = \langle A \rangle}_{\text{means average value of A}} = \sum_n a_n P(A=a_n) = \int_{-\infty}^{\infty} dx\, \psi^*(x,t)\, \hat{A}\, \psi(x,t)$$

add operator here of choice

Stationary States "motionless particles"

CM:



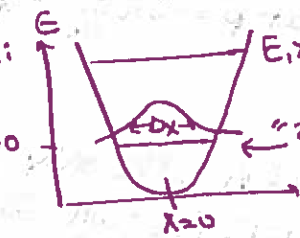
$V=\frac{1}{2}kx^2$

the only motionless state in CM

~o   ~~o

usually there's one stationary state that stays the same position

QM:



"zero point energy"

∞ many stationary states that don't change position.

Try $\Psi(x,t) = \underbrace{\varphi_n(x)}_{\text{function that depends on } x} \underbrace{e^{-i\hbar E_n t}}_{\text{function that depends on } t}$ does it satisfy the schrödinger eqn.?

is it stationary "probability is time independent"

$\rightarrow P = |\Psi(x,t)|^2 = \psi^* \psi$

$\varphi_n^*(x) \underline{e^{+i/\hbar E_n t}} \varphi_n(x) \underline{e^{-i/\hbar E_n t}}$

cancel

$= \varphi_n^*(x)\varphi_n(x) \checkmark = |\varphi(n)|$

$\hat{H}\varphi_n e^{-i/\hbar E_n t} = i\hbar \partial/\partial t \, \varphi_n(x) e^{-i\hbar E_n t} = E_n \varphi_n(x) e^{-i/\hbar E_n t}$

$= \hat{H}\varphi_n = E_n \varphi_n$ ✓ yes

Satisfies time independent if it satisfies time dependent

$\varphi_n(x)$ are eigenstates of energy $P(E=E_n)=1$

There are solutions to schrödinger eqn that are stationary states

Calculate eigenstate of the vibrating diatomic molecule

$\hat{H} = \frac{p^2}{2m} + U(x) = \frac{1}{2}\hat{p}^2 + \frac{1}{2}\hat{x}^2 = \left(-\frac{\hbar^2}{2}\frac{\partial^2}{\partial x^2} + \frac{1}{2}x^2\right)\varphi_0(x) = E_0 \varphi_0 x$

$\downarrow$ F.T.$^{-1}$

but $\hat{p} = -i\hbar \frac{\partial}{\partial x}$ & $\hat{x} = +i\hbar \frac{\partial}{\partial p} \Rightarrow \left(\frac{1}{2}p^2 - \frac{\hbar^2}{2}\frac{\partial^2}{\partial p^2}\right)\varphi_0(p) = E_0 \varphi_0(p)$

$\varphi_0(x) = \varphi_0(p)$

in this particular case

but also

$\varphi_0(x) = FT(\varphi_0(p))$

Gausian is the only function that is its own ~~function~~ Fourier Transform

$\Rightarrow \varphi_0(x) \sim e^{-ax^2}$ is a gaussian

By plugging in

$$\left(-\frac{\hbar^2}{\partial}\frac{\partial^2}{\partial x^2} + \frac{1}{2}x^2\right)e^{-ax^2} = E_0 e^{-ax^2}$$

you'll show on Hwk #8 that only

$$\psi_0 = (\pi\hbar)^{-1/4} e^{-\frac{1}{2\hbar}x^2} + E_0 = \frac{\hbar}{2}$$

can satisfy the time independent Schrödinger Eqn.

- Classical mechanics there is usally 1 stationary state
- It is possible to decouple $\psi(x,t)$ and there exist solutions where potentially infinite stationary states exist