Last time: $\hat{x}$ & $\hat{p}$ are Fourier conjugate

using $\psi(x,t)$: $\hat{x} \to x$, $\hat{p} \to -i\hbar\frac{\partial}{\partial x}$ $\Rightarrow \hat{E} = \hat{H} = \frac{\hat{p}^2}{2m} + V(x)$

$$= -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x) \quad (\text{for } \psi(x))$$

Now: Postulate 5, part 2

Equations of motion

CM: (1) $-\frac{\partial p}{\partial t} = \frac{\partial H}{\partial x}$

(2) $\frac{\partial x}{\partial t} = \frac{\partial H}{\partial p}$

$\Rightarrow$ F=ma $\to$ Verlet (How to do this on a computer)

QM: (1) $-\frac{\partial \psi^{(i)}}{\partial t} = \frac{\hat{H}}{\hbar}\psi^{(r)}(x,t)$ — real

$+\frac{\partial \psi^{(r)}}{\partial t} = \frac{\hat{H}}{\hbar}\psi^{(i)}(x,t)$ — imaginary

Derive the Schrödinger eqn.

(1) $\hat{H}\psi^{(r)} \overset{!}{=} -\hbar\frac{\partial}{\partial t}\psi^{(i)}$

(2) $i\cdot\hat{H}\psi^{(i)} = i\hbar\frac{\partial}{\partial t}\psi^{(r)}$

add

$$\hat{H}\underbrace{(\psi^{(r)} + i\psi^{(i)})}_{\text{combine}} = i\hbar\frac{\partial}{\partial t}(\psi^{(r)} + i\psi^{(i)})$$

combine

ψ has a real part and an imaginary part

$$\boxed{\hat{H}\psi = i\hbar\frac{\partial}{\partial t}\underbrace{\psi}_{\text{complex function w/ real \& imaginary part}}} \longrightarrow \text{time-dependent Schrödinger eqn.}$$

Easier to solve on a computer than by hand

SUR algorithm (quantum analog of the Verlet algo)

$$\frac{\partial}{\partial t}\psi^{(r)} = \frac{\psi^{(r)}(t+\Delta t) - \psi^{(r)}(t)}{\Delta t} \quad [\text{approximate derivative}]$$

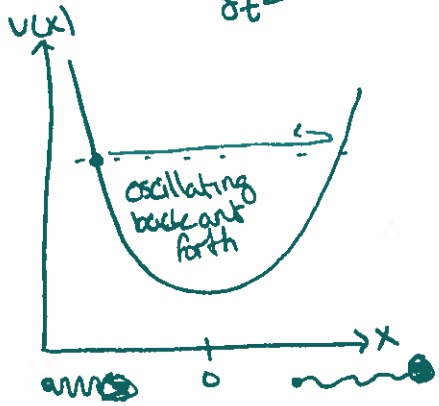
$$\Rightarrow \begin{cases} \psi^{(r)}(x, t+\Delta t) = \psi^{(r)}(x,t) + \underbrace{\frac{\Delta t}{\hbar}\hat{H}\psi^{(i)}}_{\text{use postulates}}(x,t) &] \text{ Line 1 of code} \\ \psi^{(i)}(x, t+\Delta t) = \psi^{(i)}(x,t) - \frac{\Delta t}{\hbar}\hat{H}\psi^{(r)}(x, t+\Delta t) &] \text{ Line 2 of code} \end{cases}$$

Example: Vibrating diatomic molecule = "spring"

$F = -kx \Rightarrow V(x) = -\int F dx = \frac{1}{2}kx^2$

In CM: plug into F=ma

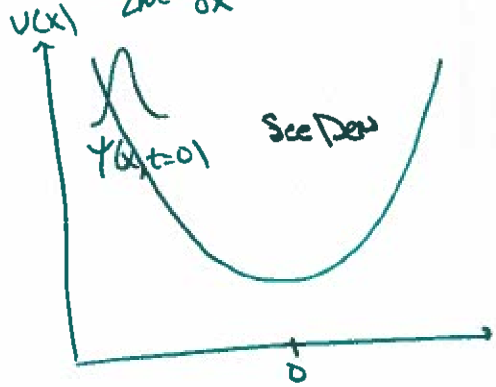
$-kx = m\frac{\partial^2 x}{\partial t^2}$



compressed (t=0)

In QM: use ~~ste~~ schrödingers eqn.

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + \frac{1}{2}kx^2\psi = i\hbar\frac{\partial\psi}{\partial t}$$



$\underbrace{\hat{E} = i\hbar\frac{\partial}{\partial t}}_{\text{similar to Fourier transform we saw previously}}$ in time "representation"

$\Rightarrow E$ & t are fourier conjugates

$\Delta E \Delta t = \frac{\hbar}{2}$; also $\Delta\omega\Delta t = 1/2$ or $\Delta(\hbar\omega)\Delta t = \frac{\hbar}{2}$

$\Rightarrow E = \hbar\omega = \frac{h}{2\pi}\underbrace{2\pi\nu}_{\omega} \Rightarrow E = h\nu$ (Planck's Law)

Energy of any particle = to $h \times$ the frequency of that particle

$$\mathcal{F}\mathcal{J}_E(\hat{H}\psi(x,t) = i\hbar\frac{\partial}{\partial t}\psi(x,t) = \hat{H}\psi(x,E) = E\psi(x,E)$$

Time-independent Schrödinger eqn

Time-dependent & independent are Fourier Transforms of one another.