Lecture 6

- No such things as $\psi(x,p)$ at the same time

Fourier principle in music and QM

Breakthrough in 1807

ω & t are Fourier conjugates w/ conjug. coeff of 1

$\Delta\omega\Delta t = 1/2$: $\psi(\omega,t)$ is ill-defined

$$\Rightarrow \psi(t) \underset{\text{inverse Fourier transform}}{\overset{\text{Fourier}}{\rightleftarrows}} \psi(\omega) \sim \underbrace{\int_{-\infty}^{\infty} dt\, \psi(t) e^{i\omega t}}_{\text{uses concept of filtering}}$$

integral > 0 when $\psi(t)$ has a frequency ω

$$\underline{\hat{\omega}\psi(t) = i\frac{\partial}{\partial t}\psi(t)} \rightleftarrows \omega \cdot \psi(\omega)$$

in the time domain $\hat{\omega}$ is a differtiand operator

Note the symmetry

can be used to simplify differtial equations

$$t \cdot \psi(t) \rightleftarrows -i\frac{\partial}{\partial \omega}\psi(\omega) = \hat{t} \cdot \psi(\omega)$$

time operator is a derivative

Quantum Mechanics

Heisenberg breakthrough (1925): postulate 1

postulate 1: x & p are conjugates w/ c.c of $\hbar$

$$\Rightarrow \frac{1}{\hbar}\Delta x\Delta p = \frac{1}{2} \text{ or } \Delta x \Delta p = \hbar/2$$

$$\psi(p) \rightleftarrows \psi(x) \sim \int_{-\infty}^{\infty} dp\, \psi(p)\, e^{i/\hbar\, x \cdot p}$$

$$\hat{x}\,\psi(p) = i\hbar\, \partial/\partial p\, \psi(p) \rightleftarrows x \cdot \psi(x)$$

$$p \cdot \psi(p) \rightleftarrows -i\hbar \frac{\partial}{\partial x} \psi(x) = \hat{p}\, \psi(x)$$

momentum operator

$\boxed{\hat{p} \equiv -i\hbar\, \partial/\partial x}$ Remember!

Note: $\hbar \ll 1$: $\hbar \approx 1.05 \times 10^{-34}\ \underbrace{m \cdot kg \frac{m}{s}}_{J \cdot s}$ $x \cdot p$

energy · time

Δx & Δp are very small, hard to detect

Must make a choice $\psi(x)$ or $\psi(p)$

$\psi(x)$: chemists (think in term of structure)

$\psi(p)$: physicist (think in current)

Postulate 5: Equation of motion

CM: $E = H = \underset{\text{kinetic}}{\frac{p^2}{2m}} + \underset{\text{potential}}{V(x)}$

QM: $\hat{H} = \frac{\hat{p}^2}{2m} + V(\hat{x})$ $\quad \underbrace{\hat{x} = x,\ \hat{p} = -i\hbar \frac{\partial}{\partial x}}_{\text{pluging}}$

$$\Rightarrow \hat{p}^2 = \left(-i\hbar \frac{\partial}{\partial x}\right)\left(-i\hbar\, \partial/\partial x\right) = \underbrace{-\hbar^2 \frac{\partial^2}{\partial x^2}}_{\text{plug into } \hat{H}}$$

$$\boxed{\Rightarrow \hat{H} = \frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x)}$$

energy operator in QM

In CM, $xp = px$ (x and p "commute")

$$\hat{x}\hat{p}\psi = x\left(-i\hbar\frac{\partial}{\partial x}\right)\psi = -i\hbar x\frac{\partial}{\partial x}\psi$$

$$\hat{p}\hat{x}\psi = \left(-i\hbar\frac{\partial}{\partial x}\right)x\psi = -i\hbar\psi - i\hbar x\frac{\partial}{\partial x}\psi$$

$$(\hat{x}\hat{p}\psi - \hat{p}\hat{x}\psi) = +i\hbar\psi \Longrightarrow \hat{x}\hat{p} - \hat{p}\hat{x} = i\hbar$$

in QM

do not commute

can't simultaneousy measure x and p