What does the fourier transform do

wave as a function of time to a function of frequency

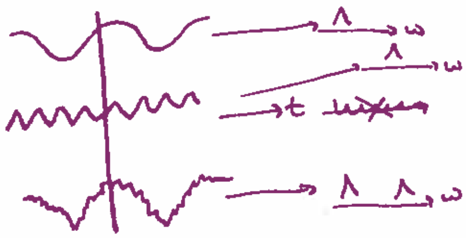
$\Rightarrow$ therefore time & frequency are not independent variables

2) use integration as a filter

multiply by $\cos \omega t$ & $\sin \omega t$ → shifted $90°$ out of phase

- takes all phases into account

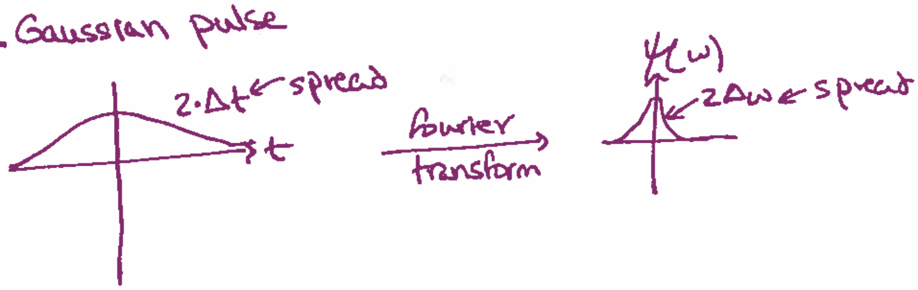
Example



Today: t (time) and ω (frequency) are fourier conjugates:

sin $\psi(\omega)$ can be calculated from $\psi(t)$ and vice versa, t and ω are not independent variables

ex. Gaussian pulse



$$\psi(\omega) \sim e^{-\Delta t^2 \omega^2} = e^{-\left(\frac{\omega}{2(1/2\Delta t)}\right)^2}$$

$$= e^{-(\omega/2\Delta\omega)^2}$$

↑ width

$\Rightarrow \Delta\omega = 1/2\Delta t$ or $\Delta\omega \cdot \Delta t = 1/2$ ← conjugation coefficient

Δt and $\Delta\omega$ are inversely proportional

Final important property of F.T derivative

$$FT\left(\frac{\partial \psi(t)}{\partial t}\right) = \int_{-\infty}^{\infty} \underbrace{\partial t \frac{\partial \psi(t)}{\partial t}}_{\partial v} \underbrace{e^{i\omega t}}_{u} = \psi(t)e^{i\omega t}\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} \partial t\, i\omega e^{i\omega t} \psi(t)$$

$v = \psi(t)$; $\partial u = i\omega e^{i\omega t} \partial t$

note:

$\int \partial v u = uv - \int \partial u v$

If $\lim_{t \to \infty} \psi(t=0) \Rightarrow FT\left(\frac{\partial \psi}{\partial t}\right) = -i\omega \int_{-\infty}^{\infty} dt e^{i\omega t} \psi(t)$

$= -i\omega \psi(\omega)$

$\boxed{\text{Turned taking a derivative into multiplication}}$

If $\psi(t)$ has a FT $\psi(\omega)$, then the derivative of $\psi(t)$ becomes

$\psi(\omega) \times -i\omega$