Analogy btwn vectors and functions (see handout)

Last time $\vec{v} \,\hat{=}\, \psi(x) \,\hat{=}\, |n\rangle \Rightarrow \vec{v} \,\hat{=}\, \sum_n c_n \vec{w}_n = \begin{pmatrix} c_1 \\ c_2 \\ \vdots \end{pmatrix} \,\hat{=}\, \psi = \sum_n c_n \varphi_n \,\hat{=}\, |\psi\rangle$
$= \sum_n c_n |n\rangle$

where $c_n = \vec{w}_n^{\,+} \cdot \vec{v}$ or $c_n = \int dx\, \varphi_n^* \psi$ or $c_n = \langle n|\psi\rangle$

Today: Operators $\hat{=}$ matrices or $\hat{A} \,\hat{=}\, \underset{\approx}{A}$

Function:
$\hat{A}\psi(x) = \chi(x)$

$\left.\begin{array}{l} \text{ex: } \hat{A}\, \frac{d}{dx} \\ \psi_1(x) = \sin x \\ \psi_2(x) = \cos x \end{array}\right\}$ $\hat{A}\psi_1 = \psi_2$, $\hat{A}\psi_2 = -\psi_1$

vector:
$\underset{\approx}{A}\vec{v} = \vec{u}$

$\left.\begin{array}{l} \text{ex } \underset{\approx}{A} = \begin{pmatrix} 0 & -1 \\ +1 & 0 \end{pmatrix} \\ \vec{w}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \; \vec{w}_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{array}\right\}$ $\underset{\approx}{A}\vec{w}_1 = \vec{w}_2$, $\underset{\approx}{A}\vec{w}_2 = -\vec{w}_1$

squiggles = matrix

<u>How to get a matrix systematicly</u>: use a complete basis

$\hat{A}\psi(x) = \chi(x) \Rightarrow \hat{A} \sum_n c_n \varphi_n(x) = \sum_n c'_n \varphi_n(x)$

$\Rightarrow \sum_n \hat{A}\varphi_n(x) c_n = \sum_n \varphi_n(x) c'_n$

$\Rightarrow \sum_n \left(\int dx\, \varphi_m^* \hat{A} \varphi_n \right) c_n = \sum_n \left(\int dx\, \varphi_m^* \varphi_n \right) c'_n$

$\Rightarrow \sum_n A_{mn} \cdot c_n = c'_m$

$\Rightarrow \begin{pmatrix} A_{11} & A_{12} & \cdots \\ A_{21} & A_{22} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} \cdot \begin{pmatrix} c_1 \\ c_2 \\ \vdots \end{pmatrix} = \begin{pmatrix} c'_1 \\ c'_2 \\ \vdots \end{pmatrix}$

$\underset{\approx}{A} \cdot \vec{v} = \vec{u}$

<u>true for any $\hat{A}$, $\psi(x)$ + $\chi(x)$</u>

- Pick a basis set φ_n
- calculate the matrix elements A_{mn} of operator $\hat{A}$
- left with matrix $\underset{\approx}{A}$

Identity operation:

vector

$\vec{v}_{\xi} = \begin{pmatrix} a \\ b \end{pmatrix} \Rightarrow$

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$

$\Rightarrow \underset{\approx}{I} = \begin{pmatrix} 1 & 0 & 0 & \cdots \\ 0 & 1 & 0 & \cdots \\ 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & \end{pmatrix}$

bracket

$|\psi\rangle = \sum_m c_m |m\rangle$

$|\psi\rangle = \sum_m \langle m|\psi\rangle |m\rangle$

$|\psi\rangle = \sum_m |m\rangle\langle m|\psi\rangle$

$|\psi\rangle = \left(\sum_m |m\rangle\langle m|\right)|\psi\rangle$

$= \hat{I}\,|\psi\rangle$

function