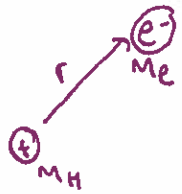
# The Hydrogen Atom




$$m = \frac{M_e M_H}{M_e + M_H} \Big\} \text{reduced mass}$$

$\approx M_e$

Similar to the rotating diatomic molecule but this time we allow r to vary:

$$\hat{H} = \frac{-\hbar^2}{2mr^2}\left(\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2} + \frac{\partial^2}{\partial\theta^2} + \cot\theta\frac{\partial}{\partial\theta}\right) \leftarrow \text{rotational}$$

$$= \underbrace{\frac{-\hbar^2}{2m}\left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}\right)}_{\text{stretching}} - \underbrace{\frac{e^2}{4\pi\varepsilon_0 r}}_{\text{coloumb attractive potential}}$$

$$= \hat{H}_{rot} + \hat{H}_r$$

$$\Rightarrow \psi_{n\ell m}(r,\theta,\varphi) = R(r) \cdot Y_{\ell m_\ell}(\theta,\varphi)$$

Check differential eqn for r

$$\Rightarrow (\hat{H}_{rot} + \hat{H}_r) R(r) \cdot Y_{\ell m} = E_{n\ell m} \cdot \underbrace{R(r)}_{\text{wavefunction of } r} \cdot Y_{\ell m}$$

$$\left(+\frac{\hbar^2 \ell(\ell+1)}{2mr^2} + \hat{H}_r\right) R(r) \cdot Y_{\ell m} = \text{''}$$

rewrite

$$\left\{ \underbrace{\frac{-\hbar^2}{2m}\left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}\right)}_{\text{kinetic}} \underbrace{- \underbrace{\frac{e^2}{4\pi\varepsilon_0 r}}_{\text{coloumb potential energy}} + \overbrace{\frac{\hbar^2 \ell(\ell+1)}{2mr^2}}^{\text{rotational potential energy}}}_{\text{effective potential energy}} \right\} R_{n\ell}(r) = E_{n\ell m} R_{n\ell}(r)$$


Try $R_{10}(r) \sim e^{-kr}$ $=$ $(n=1, \ell=0)$

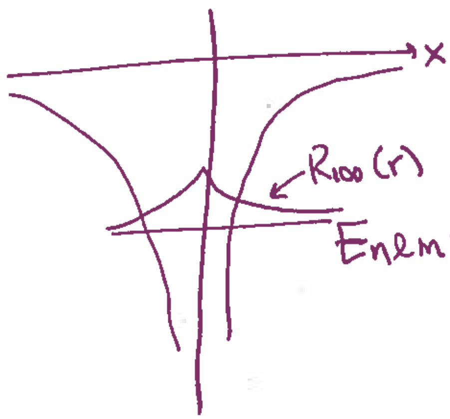
$$\Rightarrow \hat{H}_r e^{-kr} = -\frac{\hbar^2}{2m}\left((-k)^2 e^{-kr} + \frac{2k}{r} e^{-kr}\right) + \frac{e^2 e^{-kr}}{4\pi\varepsilon_0 r} = E_{n00}\, e^{-kr}$$

$$\Rightarrow E_{100} = -\frac{\hbar^2}{2m}k^2 + \frac{1}{r}\underbrace{\left(\frac{e^2}{4\pi\varepsilon_0} - \frac{\hbar^2 k}{m}\right)}_{\text{needs to be 0 since } E_{n00} \text{ is an eigenvalue, not } f(r)}$$

$$\Rightarrow k = \frac{1}{a_0} = \frac{me^2}{4\pi\varepsilon_0\hbar^2} \Rightarrow E_{100} = -\frac{me^4}{32\pi\varepsilon_0^2\hbar^2} \approx -2.1798\times10^{-18}\,\text{J}$$

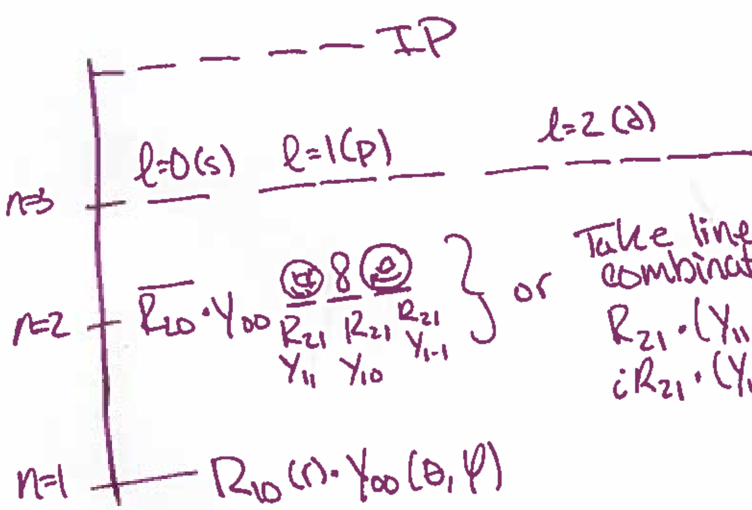
"bohr radius"

electron is orbiting (negative value)




$E_{nlm} = E_{100} = E_{1s}$ agrees w/ ionization potential of H to 12 sig figs

raising and lowering operators could be used to calculate excited states




or Take linear combination

$R_{21}\cdot(Y_{11} + Y_{1-1})$

$iR_{21}\cdot(Y_{11} - Y_{1-1})$