Last time: 3-D rotation, $\hat{H}_{rot} = \frac{\hat{L}^2}{2\mu R_r^2}$

$\hat{H} \cdot Y_{\ell m}(\theta,\varphi) = E_{rot} \cdot Y_{\ell m}(\theta,\varphi)$

rotational wave function

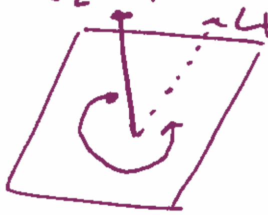
$E_{rot} = \frac{\hbar^2 \ell(\ell+1)}{2mr^2}$ $\ell = 0,1,2..$ $Y_{\ell m}(\theta,\varphi) = P_{\ell m} \cdot e^{iM\varphi}$

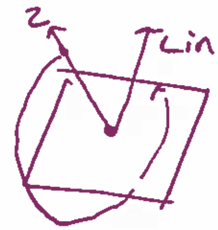
why does E_{rot} not depend on M

$L_z = \hbar M \neq 0$

$L_{tot} = \hbar\sqrt{\ell(\ell+1)}$

direct consequence of uncertainty





L in X-Y plane
$L_z = \hbar M = 0$

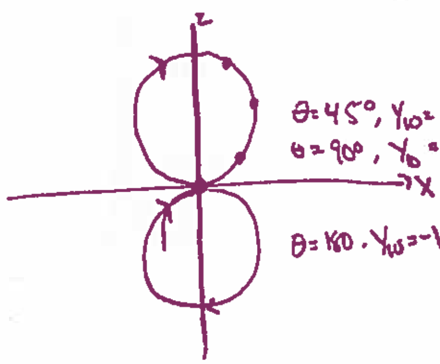
Do I care about orientation or the $\boxed{\text{velocity}}$

Magnitude of angular momentum tells us what the energy is
(P_x, P_y, P_z are at the same energy, but different orientation)

Plotting rotational wavefunctions: Polar plot

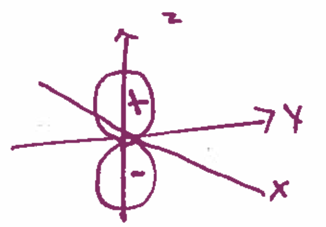
$R = |Y_{\ell m}(\theta,\varphi)|$ gives magnitude of function

ex: $Y_{10}(\theta,\varphi) \sim \cos\theta\, e^{i0\varphi} = \cos\theta$



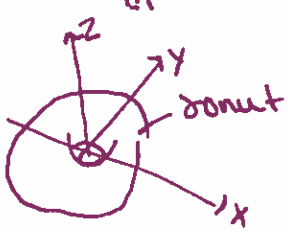
$\theta = 45°$, $Y_{10} = .7$
$\theta = 90°$, $Y_{10} = 0$

$\theta = 180$, $Y_{10} = -1$



$Y_{10} = 8 = 8 = P_z$

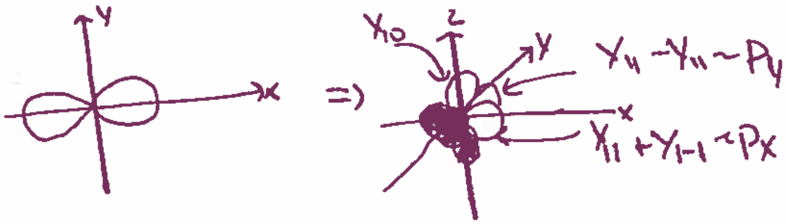
ex: $Y_{1\pm1} \sim \sin\theta\, e^{\pm i\varphi} \Rightarrow |Y_{11}|^2 = |Y_{1-1}|^2 = \sin^2\theta$



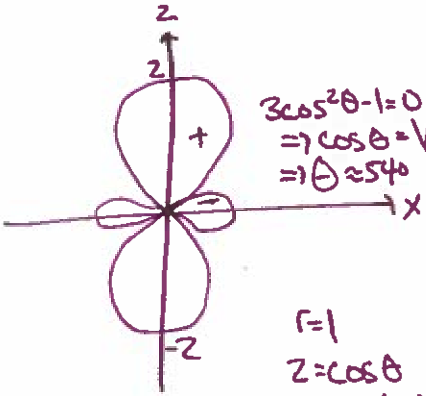
$Y_{1\pm1}$ are degenerate $E_{1\pm1} = \frac{\hbar^2 1 \cdot 2}{2mr^2} = \frac{\hbar^2}{2mr^2}$

$\Rightarrow Y_{11} \pm Y_{1-1}$ are also eigen functions

$= \sin\theta\,(e^{i\varphi} + e^{-i\varphi}) \sim \sin\theta\cos\varphi$



$Y_{20}(\theta,\varphi) \sim 3\cos^2\theta - 1$



$3\cos^2\theta - 1 = 0$

$\Rightarrow \cos\theta = \sqrt{1/3}$

$\Rightarrow \theta \approx 54°$

$Y_{20} \hat{=}$ 

$= d_{2z^2-x^2-y^2}$

$= d_{z^2}$

$r = 1$

$z = \cos\theta$

$y = \sin\theta\sin\varphi$

$x = \sin\theta\cos\varphi$

$2z^2 - x^2 - y^2$

$(2\cos^2\theta - \sin^2\theta(\sin^2\varphi +$

$(2\cos^2\theta - (1-\cos^2\theta)) = (3\cos^2\theta - 1)$

CdSe ~~Croot~~ quantum dots

CM

CdSe is a spherical box

QM

$\psi(\theta,\varphi,r) = R(r) \cdot P_{lm}(\theta) \cdot e^{im\varphi}$

ex. $P_{10}(\theta)e^{i0\varphi} = P_z$ state $=$