Last time: Problems in 2 or more dimensions

2-D or 3-D box $\hat{H} = \frac{-\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)$

$= \hat{H}_x + \hat{H}_y + \hat{H}_z$ } sum

$\Rightarrow \psi(x,y,z) = \psi(x)\cdot\psi(y)\cdot\psi(z)$ } product

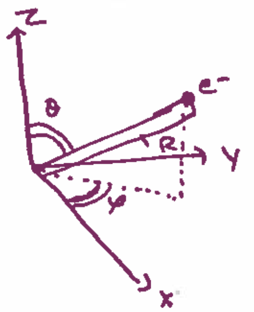
convinient for more advanced types of particles

ex- rotating-vibrating molecule

$\hat{H} = \hat{H}_{rot}(\varphi) + \hat{H}_{vib}(r)$ } sum

$\psi(\varphi, r) = \psi(\varphi)\cdot\psi(r)$ } product

Today: rotation in 3D




θ and φ may be flipped in mathematics books

$\hat{H} = \frac{-\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right) + V(x,y,z)$

change to polar coordinates

$r = \sqrt{x^2+y^2+z^2}$

$\varphi = \tan^{-1}(y/x)$

$\theta = \tan^{-1}(\sqrt{x^2+y^2}/z)$

$\hat{H}(\varphi,\theta,r) = \frac{-\hbar^2}{2mr^2}\left(\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}+\frac{\partial^2}{\partial\theta^2} + \frac{\cos\theta}{\sin\theta}\frac{\partial}{\partial\theta}\right) +$

$\left\{ -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}\right) + V(r)\right.$

looks the same as the rotating-vibrating molecule

interested in coloumb potential (only depends on r)

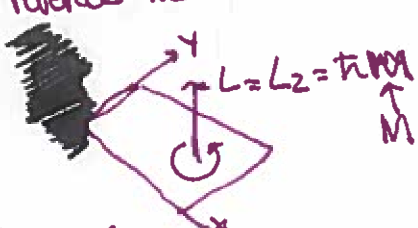
$\hat{H} = H_{rot}(\theta,\varphi) + \hat{H}(r)$

$\psi = \psi(\theta,\varphi)\cdot\psi_r(r)$

As in 2D, $\hat{H}_{rot} = \frac{\hat{L}^2}{2mR^2}$

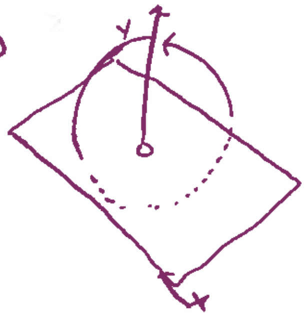
[redacted]

Difference between 2D and 3D rotation




$\hat{H}_{rot} = \frac{\hat{L}_z^2}{2mr^2} = \frac{-\hbar^2}{2mR^2}\frac{\partial^2}{\partial\varphi^2}$ $\quad \psi_{rot}(\varphi) \sim e^{iM\varphi}$, $L_z = \hbar M$

3D



Angular Momentum can point anywhere in 3D

In the case above $L_z = \hbar M = 0$

but "$l$" $\neq 0$

($l$ = total angular momentum)

The solutions for $\hat{H}_{rot} \cdot \psi_{rot}(\theta, \varphi) = E_{lM} Y_{lM}(\theta, \varphi)$ can be derived using raising and lowering operators, but... not going through that again

$$\frac{L^2}{2\mu r^2} Y_{lm}(\theta,\varphi) = \underbrace{\frac{\hbar^2 l(l+1)}{2\mu r^2}}_{E_{rot}} Y_{lm}(\theta,\varphi) \quad l = 0, 1, \ldots$$

We have two coordinates $\theta, \varphi \Rightarrow$ two quantum #s

$l$ and $M$ ← z-axis angular momentum $M = 0, \pm 1, \pm 2$

↑

total angular momentum

$l = 0, 1, 2$

$|M| \leq l$

The length of a vector can't be longer than it's z-axis component

Lecture on Monday → what these wavefunctions look like