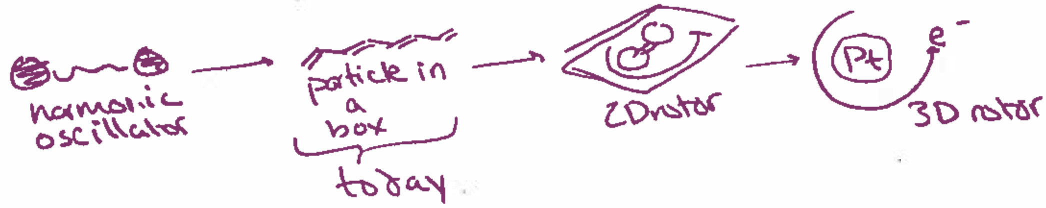
Lecture 12

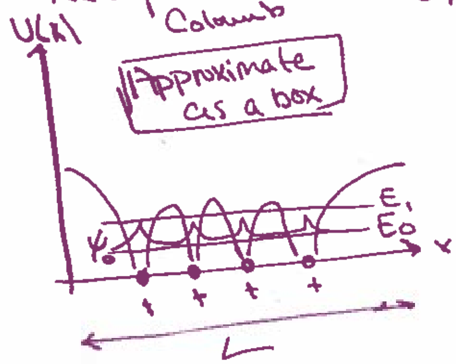
Please see summary sheet from previous lecture

future: calculate the properties of model systems

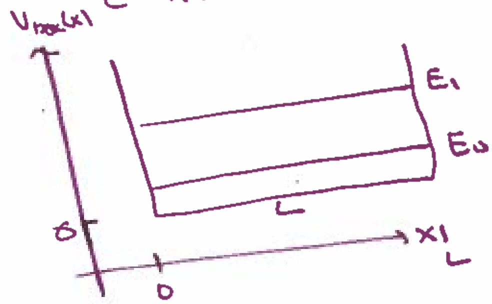


- e⁻ provide the "spring" between atoms

- Real potential energy:



simple model
e⁻ in a box



$$\frac{-\hbar^2}{2m_e}\frac{\partial^2\psi}{\partial x^2} + U_{box}(x)\psi = E_n\psi$$

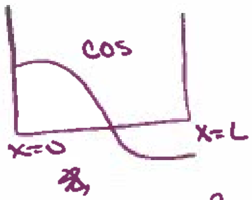
But $U_{box}=0$ for $0<x<L$ and $\underbrace{U_{box}=\infty \text{ outside}}_{e^- \text{ stuck in the box}}$

Solve the schrödinger eqn.

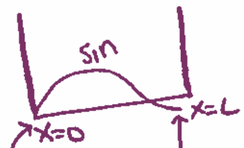
$$\frac{-\hbar^2}{2m_e}\frac{\partial^2\psi}{\partial x^2} + 0\cdot\psi = E_n\psi, \quad \begin{cases}\psi(x\le 0)=0\\ \psi(x\ge L)=0\end{cases}$$

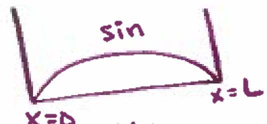
Because the probability of finding the particle outside the box is 0

$$\Rightarrow \frac{\partial^2 \psi_n}{\partial x^2} = -\frac{2m_e}{\hbar^2} E_n \psi_n \Rightarrow \begin{array}{l} \psi_n(x) = \sin\sqrt{\frac{2m_e E_n}{\hbar^2}}\,x \\ \psi_n(x) = \cos\sqrt{\frac{2m_e E_n}{\hbar^2}}\,x \end{array}$$



Doesn't satisfy boundary conditions

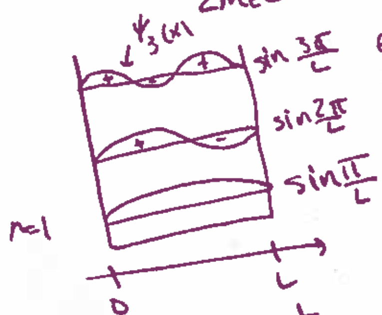




$$\Rightarrow \psi_n(L) = 0 = \sin\sqrt{\frac{2m_e E_n}{\hbar^2}}\,L \Rightarrow \sqrt{\frac{2m_e E_n}{\hbar^2}}\,L = n\pi$$

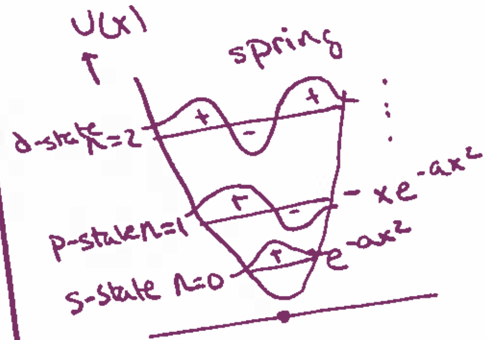
$n = 1, 2, 3 \ldots$

$$\Rightarrow E_n = \frac{\hbar^2 n^2 \pi^2}{2m_e L^2}$$



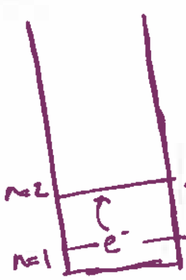
energy ↑ w/ the square of n. spacing increases

n=0 is not a solution



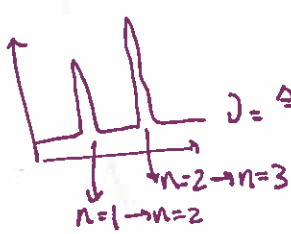
They look similar

Transitions:



$$\Delta E = \frac{h^2}{8mL^2}(2^2 - 1^2)$$

Energy conservation:
only energy packets (heat, light ...) that match an energy level difference can be absorbed



$$\nu = \frac{\Delta E}{h}$$