

Last time: a vibrating molecule has many } eigenstates
 } energy levels

Curvature of higher energy wavefunctions increase
 curvature $\uparrow \Rightarrow$ energy \uparrow
 function is spread out over more space \Rightarrow potential energy \uparrow

$$\psi_0 = G(x)$$

$$\psi_1 \sim \hat{a}^+ \psi_0 \sim x \cdot G$$

$$\psi_2 \sim \hat{a}^+ \psi_1 \sim \hat{a}^{+2} \psi_0$$

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2} k x^2$$

$$= \hbar \omega (\hat{a} + \frac{1}{2})$$

Math Trick:

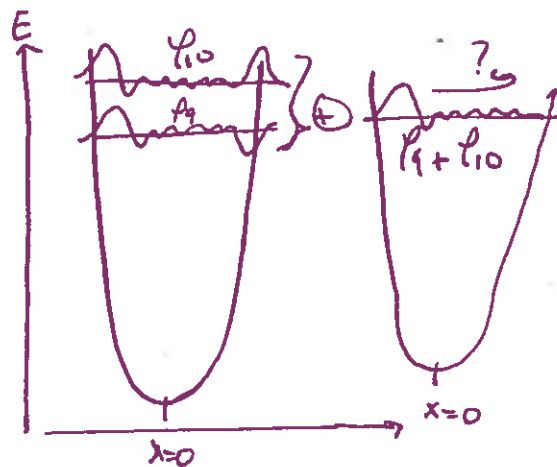
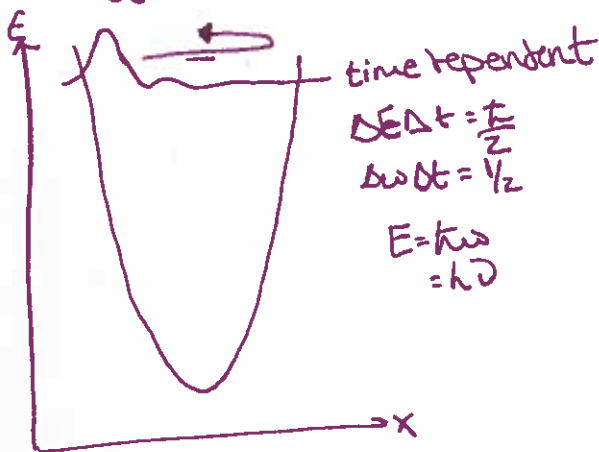
$$\int \psi_n^*(x) \psi_m(x) dx = 1$$

$$\text{if } n=m; = 1$$

$$\text{if } n \neq m; \text{ "orthogonal"}$$

Equivalence of:

$$\hat{H} \psi = i \hbar \frac{\partial}{\partial t} \psi \xrightarrow{FT} \hat{H} \psi_n = E_n \psi_n$$



Equivalent, $\psi(x,t)$ can always be written as a sum of eigenstates

$$\psi(x,t) = \sum_m C_m \cdot \psi_m(x) e^{-i/\hbar E_m t}$$

$$1) \hat{H} \psi = \sum_m C_m \hat{H} \psi_m e^{-i/\hbar E_m t}$$

$$= \sum_m C_m E_m \psi_m e^{-i/\hbar E_m t}$$

$$2) i \hbar \frac{\partial}{\partial t} \psi = \sum_m C_m \psi_m i \hbar \frac{\partial}{\partial t} e^{-i/\hbar E_m t}$$

$$= \sum_m C_m \psi_m E_m e^{-i/\hbar E_m t}$$

The same!
 satisfying time dependent Schrödinger equation

But what are the coefficients

$$\psi = \sum_n c_n \psi_n e^{-i/\hbar E_n t} \quad \rightarrow$$

$$\psi_n^*(x) \psi(x,0) = \sum_m c_m \psi_m^* \psi_m \frac{e^{-i/\hbar E_m t}}{1}$$

$$\int_{-\infty}^{\infty} dx \psi_n^* \psi(x,0) = \sum_m c_m \int_{-\infty}^{\infty} dx \psi_n^* \psi_m$$

$$= c_n$$

If we know $\psi(x,t=0)$ our "initial condition" we can calc. all the c_m and then $\psi(x,t)$ at any time!

Spectroscopy: energy levels mean only certain frequencies of light can be absorbed

