

# 442 Class Notes

- Zohaib is answering questions on chat or relaying to prof. Gruebele
- Please see syllabus for details
- Moodle is for grading only
- HW solutions are provided beforehand

## Material

Postulates:  $\rightarrow$  There are 6

### ① fundamental observables

- in CM these are  $x$  &  $p$   $\leftarrow$  momentum

$\uparrow$   
position

- in QM these are  $\hat{x}$  or  $\hat{p}$  one can be used to determine the other  
 $\underbrace{\hspace{2cm}}_{\text{not independent}}$

### ② How to describe state of system

- in CM  $x(t)$  and  $p(t)$  are sufficient

- in QM  $\rightarrow$  state is defined by a wavefunction  $\psi(x,t)$

### ③ Derived observables

- in CM dipole moment =  $q \cdot x$   $\leftarrow$  note the similarity!

- in QM dipole moment ( $\hat{\mu}$ ) =  $q \cdot \hat{x}$   $\rightarrow$  this utilizes operators  
operators "operate" on a function

### ④ Measurement of observables

- in CM position and momentum ~~can~~ (and must) be measured to arbitrary precision

- the uncertainty principle:  $\Delta x \Delta p = \frac{\hbar}{2}$   $\rightarrow$  very small number  
 $\downarrow$   
can't be measured to arbitrary values  
can't measure on the scale of classical mechanics

### ⑤ Equation of motion

CM  $\rightarrow$  hamilton's eqn

QM  $\rightarrow$  schrodinger's equation

### ⑥ Particles

- CM  $\rightarrow$  two different objects can be distinguished

- QM  $\rightarrow$  two electrons can be completely indistinguishable  
- rationalizes the Pauli exclusion principle

From these 6 equations, we can derive most of quantum mechanics.

$$E = H = \frac{p^2}{2m} + V(x)$$

↑ Kinetic energy  
↑ Potential energy

a)  $\frac{\partial x}{\partial t} = \frac{\partial H}{\partial p}$

b)  $\frac{\partial p}{\partial t} = -\frac{\partial H}{\partial x}$

a)  $\frac{\partial x}{\partial t} = \frac{p}{m} \Rightarrow p = m \frac{\partial x}{\partial t}$

b)  $-\frac{\partial p}{\partial t} = \frac{\partial V}{\partial x} = -F \Rightarrow \frac{\partial p}{\partial t} = F = m \frac{\partial^2 x}{\partial t^2}$

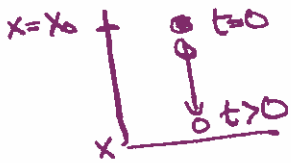
↑ Force (by definition)

↑ this is the acceleration

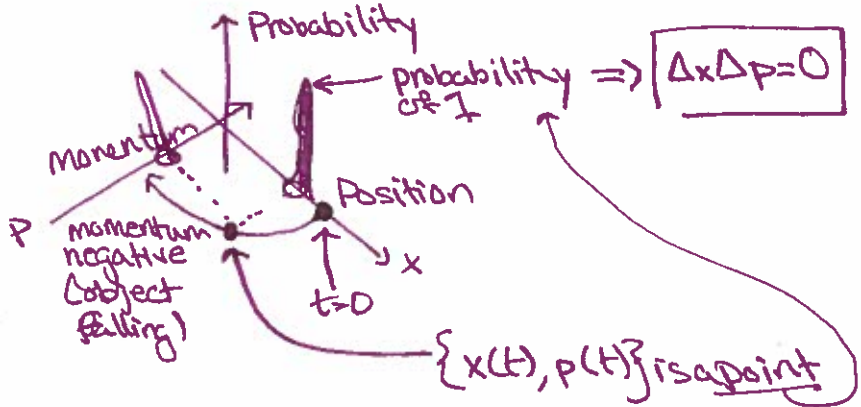
$= ma$

In Quantum mechanics, there are directly analogous set of equations.

Consider a falling electron in a vacuum chamber in CM

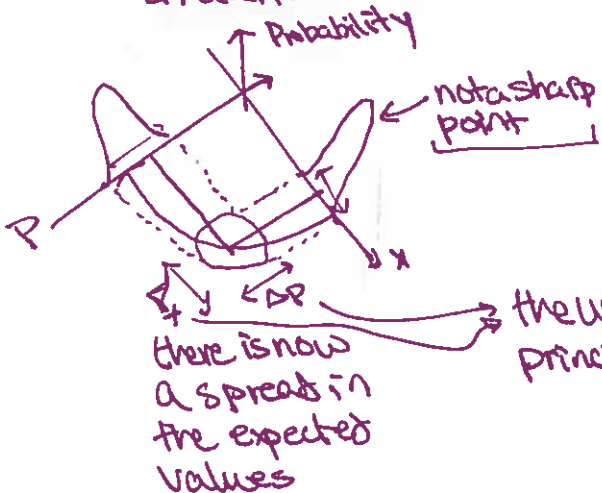


another way to plot



X-P plot = phase space

Now consider the same situation in QM  $\Rightarrow$  uncertainty principle



$P(x,t) = |\psi(x,t)|^2$  describes the state at  $t$   
we'll prove this later

In QM  $x$  or  $p$  can be measured bc  $x$  or  $p$  is sufficient to describe the state

$\psi(x,t)$  or  $\psi(p,t)$

In this sense, it's not really an "uncertainty" principle

Handwritten notes and diagrams illustrating quantum mechanics concepts:

- Wavefunctions:**  $\psi(x,t)$  and  $\psi(p,t)$  are shown as functions of position and momentum respectively.
- Measurement:** Notes discuss how measuring  $x$  or  $p$  collapses the wavefunction.
- Uncertainty Principle:** A diagram shows two probability distributions on the  $x$ -axis. One is narrow (high uncertainty in  $p$ ), and the other is wide (low uncertainty in  $p$ ), illustrating the trade-off between  $\Delta x$  and  $\Delta p$ .
- Commutators:**  $[x, p] = i\hbar$  is noted, explaining why both cannot be known simultaneously.
- Wavepacket:** A diagram shows a localized wavepacket in position space, composed of many plane waves in momentum space.
- Fourier Transform:** The relationship between  $\psi(x)$  and  $\psi(p)$  is shown as a Fourier transform pair.
- Probability Density:**  $|\psi(x)|^2$  and  $|\psi(p)|^2$  are noted as the probability densities for finding the particle at a certain position or momentum.