442 Class Notes

- Zohaib is answering questions on chat or relaying to prof. Gruebele
- Please see syllabus for details
- Moodle is for grading only
- HW solutions are provided beforehand

Material

Postulates: ⟶ There are 6

1. fundamental observables
   - in CM these are x & p ← momentum
     position
   - in Q these are $\underbrace{\hat{x}\ \boxed{\text{or}}\ \hat{p}}_{\text{not independent}}$ one can be used to determine the other
2. How to describe state of system
   - in x(t) and p(t) are sufficient
   - in QM → state is defined by a wavefunction $\psi(x,t)$
3. Derived observables
   - in CM dipole moment = $q \cdot x$ note the similarity!
   - in QM dipole moment $(\hat{\mu}) = q \cdot \hat{x}$ → this utilizes operators
     operators "operate" on a function
4. Measurement of observables
   - in CM position and momentum can ~~can~~ (and must) be measured to arbitrary precision
   - the uncertainty principle: $\underbrace{\Delta x \Delta p}_{\text{can't be measured to arbitrary values}} = \hbar/2$ → very small number ↓ can't measure on the scale of classical mechanics
5. Equation of motion
   - CM → hamilton's eqn
   - QM → schrödinger's equation
6. Particles
   - CM → two different objects can be distinguished
   - QM → two electrons can be completely indistinguishable
     - rationalizes the Pauli exclusion principle

From these 6 equations, we can define most of quantum mechanics.

$$E = H = \frac{p^2}{2m} + V(x)$$

$\frac{p^2}{2m}$ ↑ kinetic energy; $V(x)$ ↑ potential energy

a) $\frac{\partial x}{\partial t} = \frac{\partial H}{\partial p}$

b) $-\frac{\partial p}{\partial t} = \frac{\partial H}{\partial x}$

a) $\frac{\partial x}{\partial t} = \frac{p}{m} \Rightarrow p = m\frac{\partial x}{\partial t}$

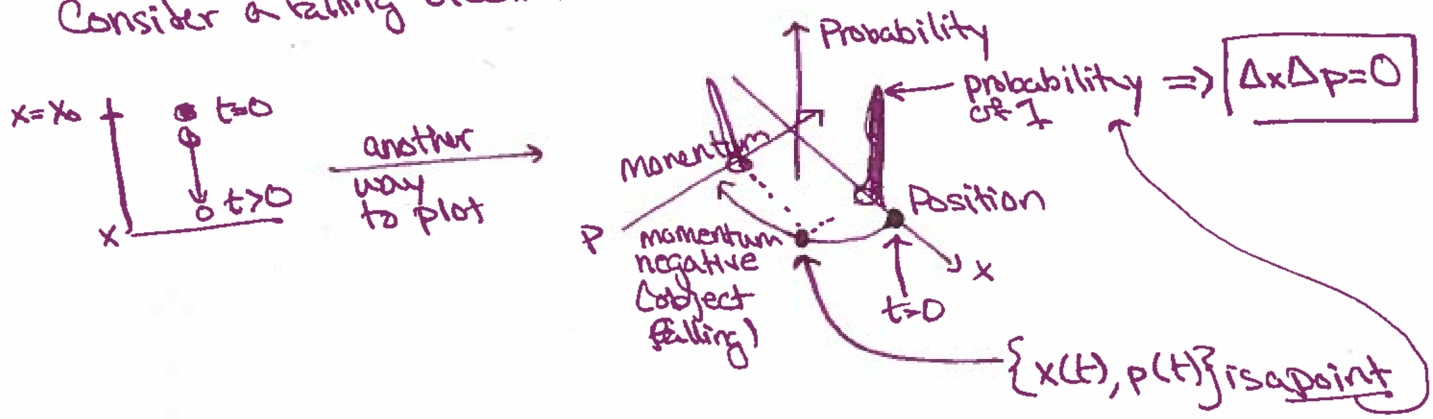
b) $-\frac{\partial p}{\partial t} = \frac{\partial V}{\partial x} = -F \Rightarrow \frac{\partial p}{\partial t} = F = m\underbrace{\frac{\partial^2 x}{\partial t^2}}_{\text{this is the acceleration}}$

F ↑ force (by definition)

$\boxed{= ma}$

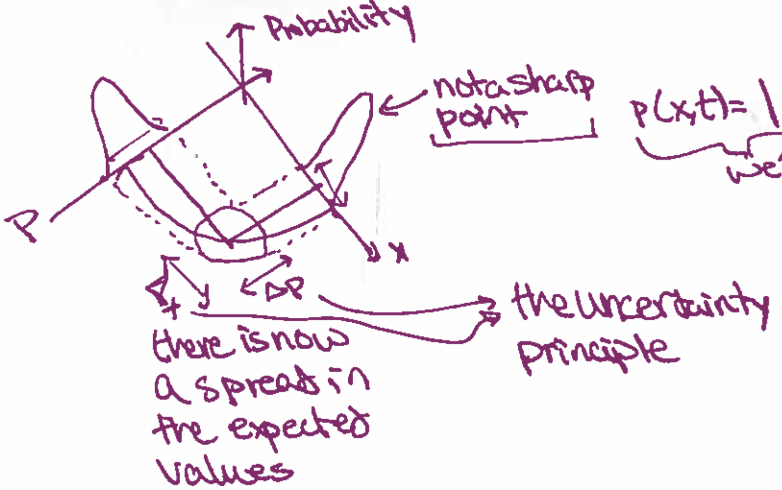
In Quantum mechanics, there are directly analogous set of equations.

Consider a falling electron in a vacuum chamber in CM



X-P plot = phase space

Now consider the same situation in QM ⇒ uncertainty principle



$\underbrace{P(x,t) = |\Psi(x,t)|^2}_{\text{we'll prove this later}}$ describes the state at t

In QM x or p can be measured b/c x or p is sufficient to describe the state

$$\psi(x,t) \text{ or } \psi(p,t)$$

In this sense, it's not really an "uncertainty" principle