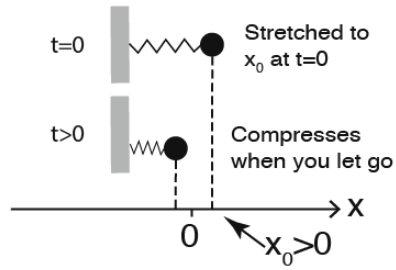
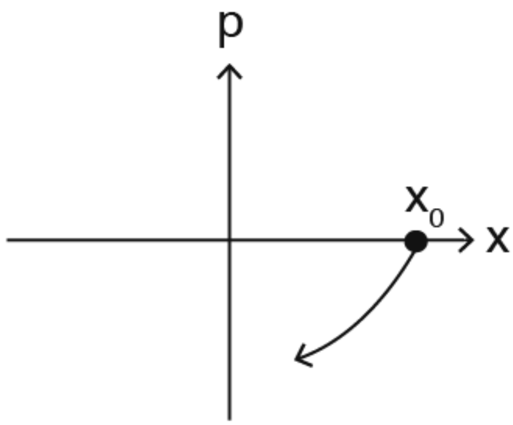
CHE 442A
Fall 2015

Exam 1



1a. (5 pts) On an x-p plot, draw the trajectory of a ball vibrating on a spring, shown on the right. Draw the trajectory for the ball starting at $x_0>0$ and p=0. (x=0 is the equilibrium position of the spring.) **Draw one cycle of the vibration**. We started the plot for you below:



b. (10 pts) In quantum mechanics, a particle has a spread Δx and Δp associated with it. **Draw a quantum particle** on the same plot when its average momentum is p=0, and label Δx and Δp. **What is the "area"** of the quantum particle on the x-p plot? **Write down** the formula for the "uncertainty" principle involved.

2. A wavefunction is given by $\Psi(x) = \frac{1}{\sqrt{\pi}} e^{-x^2} e^{ikx}, k = 10\ m^{-1}$

a. (5 pts) **What is its numerical value** at x=1, written in the form $\Psi(x) = a + i\, b$?
b. (5 pts) **What is** the complex conjugate of the number $a + i\, b$ in general, and **what is** the complex conjugate of the wavefunction in (a) numerically?
c. (5 pts) **Show that** the wavefunction times its complex conjugate is a real number.

3. A pianist plays a scale of notes in a Chopin Ballade *prestissimo* (200 notes per second). He hits a high "A" (1760 Hz) for $1/200^{th}$ of a second.

a. (5 pts) As a percentage of frequency, **what is** the spread in frequency of the high "A"?

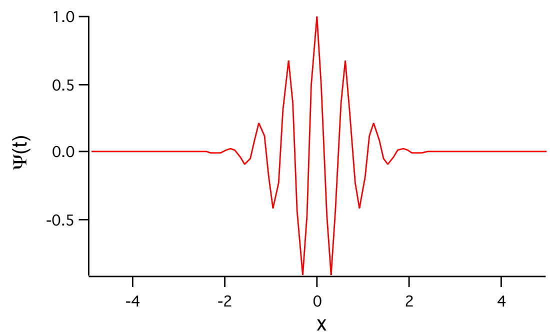
b. (5 pts) The next whole note on the piano is B, a factor of $2^{(2/12)} \approx 1.122$ higher in frequency than A. **What is** the frequency of B? Can A and B **be distinguished** when played $1/200^{th}$ second each?

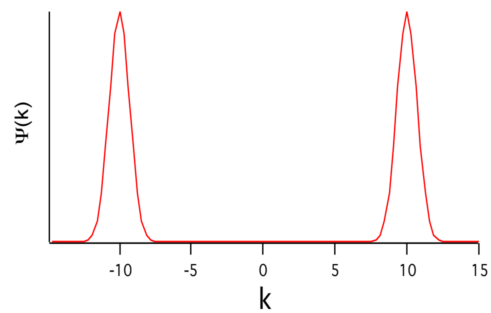
c. (5 pts) If a piano-playing robot were built that plays the passage at $1/1000^{th}$ second per note, **could** individual notes A, B, C… in a scale be distinguished?

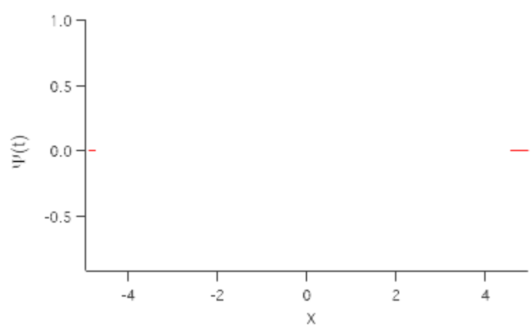
CHE 442A
Fall 2015

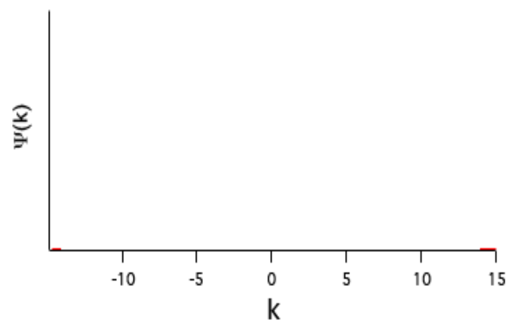
4. Below is shown a plot of the real part of the wavefunction $\Psi(x)$ in problem 2, and its Fourier transform $\Psi(k)$.

a. (10 pts) **Sketch roughly** the function $\Psi(x) = \frac{1}{\sqrt{2\pi}} e^{-(x/2)^2} \cos 10x$ below it, and also **sketch its Fourier transform**. (<u>The function in 4a. has x/2 in the Gaussian instead of *x*.</u>)









5. According to postulate (5), the wavefunction satisfies the two equations

$$+\frac{\partial \Psi^{(r)}}{\partial t} = \frac{\hat{H}}{\hbar}\Psi^{(i)}$$
$$-\frac{\partial \Psi^{(i)}}{\partial t} = \frac{\hat{H}}{\hbar}\Psi^{(r)},$$

from which we derived the time-dependent Schrödinger equation in class.

a. (5 pts) Replace the time derivative in both equations by a difference involving Ψ□□□□□□□□(t+Δt), Ψ□□□□□□□□□(t) and Δt, and **write down** the resulting two finite-difference equations.

b. (5 pts) Solve the two equations for Ψ(t+Δt) to **obtain**

Ψ□□□(t+Δt) = (1)
Ψ□i□(t+Δt) = (2)

with explicit equations in place of (1) and (2).

c. (5 pts) Now assume we get even lazier than baby calculus and on the computer, after we calculate Ψ□□□(t+Δt) from Ψ□□□(t) in the first line (1), we store the new value Ψ□□□(t+Δt) ON TOP OF the old value Ψ□□□(t), erasing it. **How does that modify** (2)?

Congrats, you have derived the SUR algorithm for moving the wavefunction forward in time on a computer.