Heisenberg's discovery: x and p are Fourier conjugate

-Thus they are not independent variables
-**All Fourier-transform properties hold,** e.g.

- -No function $\Psi(x,p)$ exists, only $\Psi(x)$ or $\Psi(p)$
- -$\Psi(p)$ and $\Psi(x)$ are related by Fourier transform
- -the width of $\Psi(p)$ and $\Psi(x)$ are related by $\Delta x \Delta p = \hbar/2$
- -$p\Psi(p)$ upon Fourier transform becomes $p\Psi(x) = -i\hbar\partial/\partial x\,\Psi(x)$
- -$x\hat{p} \neq \hat{p}x$ because order of differentiation matters!

Thus if we want to use $\Psi(x)$, p is no longer just a number (multiplicative operator) but a derivative with respect to x (differentiation operator) because the number "p" cannot be defined simultaneously with the number "x".

Vibrating molecule