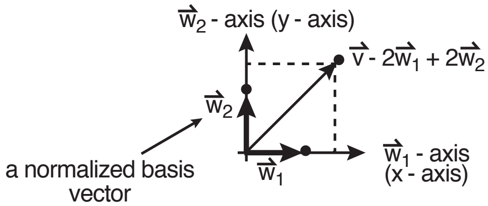
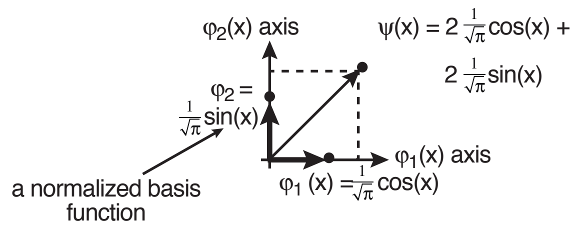
Last time: Normalizable functions and vectors behave analogously:

Ordinary vector space:



vectors point to (x,y) coords.

Hilbert space:



vectors point to functions

Any vector in the vector space can be written as a sum over basis vectors:

$$\vec{v} = c_1\vec{w}_1 + c_2\vec{w}_2 + \cdots = \sum_n c_n\vec{w}_n$$

Vectors $\vec{w}_1=\hat{x}$, $\vec{w}_2=\hat{y}$, etc. form a complete basis of orthonormal vectors!

$$c_n = \vec{w}_n^\dagger \bullet \vec{v}$$

Any function in the Hilbert space can be written as a sum over basis functions:

$$\Psi(x) = c_1\varphi_1(x) + c_2\varphi_2(x) + \cdots = \sum_n c_n\varphi_n(x)$$

Solutions of an eigenvalue differential equation form a complete basis of orthonormal functions!

$$c_n = \int dx\, \varphi_n^*(x)\Psi(x)$$

Example vectors and functions for this particular case of vector and Hilbert space:

$$\vec{w}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \vec{w}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \vec{v} = \begin{pmatrix} \sqrt{\pi} \\ \sqrt{\pi} \end{pmatrix} \qquad \vec{u} = \sqrt{\pi}\begin{pmatrix} 1 \\ i \end{pmatrix}$$

$$\Psi(x) = \varphi_1(x) = \frac{1}{\sqrt{\pi}}\cos x$$

$$\Psi(x) = \varphi_2(x) = \frac{1}{\sqrt{\pi}}\sin x$$

$$\Psi(x) = \sqrt{\pi}\,\varphi_1(x) + \sqrt{\pi}\,\varphi_2(x) = \cos x + \sin x$$

$$\chi(x) = e^{ix}$$

Note: many possible Hilbert spaces of different pairs of functions correspond to the 2-D vector space. (And of course this is also true for higher-dimensional vector and Hilbert spaces). For example, the functions sin(2x) and cos(2x) also form a 2-D Hilbert space “isomorphic” with the 2-D vector space. The functions sin(Mx) and cos(Mx) would form an ∞ dimensional Hilbert space based on the solutions of the rotating molecule Schrödinger equation (=eigenvalue differential equation). ANY normalizable function of angle can be expressed in terms of them.