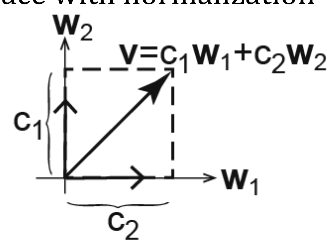
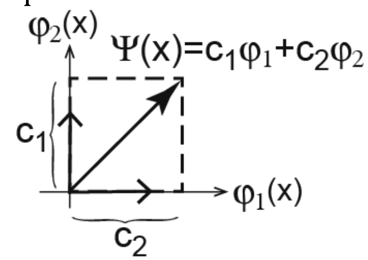
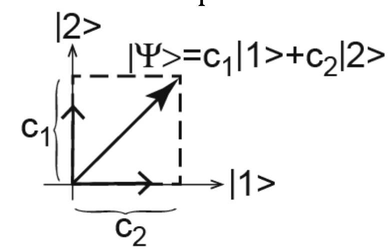
Analogy between vectors and functions

	Vectors	Functions	Dirac vector-function notation
1	Vector **v** or $\mathbf{w}_n$	Function $\Psi(x)$ or $\varphi_n(x)$	ket $\vert\Psi\rangle$ or $\vert n\rangle$
2	Dot product $\mathbf{v}^\dagger\cdot\mathbf{w}_n$ = number $\mathbf{v}^\dagger\cdot\mathbf{v}=1$ means vector **v** is normalized $\mathbf{w}_n^\dagger\cdot\mathbf{w}_m=0$ orthogonal vectors	Overlap integral $\int dx\, \Psi^*(x)\varphi_n(x)$ = number $\int dx\, \Psi^*(x)\, \Psi(x)=1$ means function $\Psi(x)$ is normalized $\int dx\, \varphi_n^*(x)\, \varphi_m(x)=0$ orthogonal functions	bracket $\langle\Psi\vert n\rangle$ = number $\langle\Psi\vert\Psi\rangle=1$ means ket $\vert\Psi\rangle$ is normalized $\langle n\vert m\rangle=0$ orthogonal bracket
3	Vector space with normalization 	Function space with normalization… 	…also called Hilbert space 
4	Complete basis set: All orthonormal vectors $\mathbf{w}_n$ that span the vector space (usually 3-D); any vector can be expressed as a sum of them.	Complete basis set: All orthonormal functions $\varphi_n(x)$ that solve an eigenvalue equation $\hat{A}\,\varphi_n(x) = a_n\,\varphi_n(x)$; any function can be expressed as a sum of them.	Complete basis set: All orthonormal kets that solve the eigenvalue equation $\hat{A}\,\vert n\rangle = a_n\,\vert n\rangle$; any ket can be expressed as a sum of them.
5	Getting coefficient c_n: $\mathbf{w}_n^\dagger\cdot\mathbf{v} = c_n$	Getting coefficient c_n: $\int dx\, \varphi_n^*(x)\, \Psi(x) = c_n$	Getting coefficient c_n: $\langle n\vert\Psi\rangle = c_n$
6	The dual space element: $\mathbf{w}_n^\dagger\cdot$	The dual space element: $\int dx\, \varphi_n^*(x)$__	The dual space element: (called 'bra') $\langle n\vert$
7	Operator: transforms a vector into another: **u** = A **v** They are called "matrices"	Operator: transforms a function into another: $\chi(x) = \hat{A}\,\Psi(x)$ They are called "operators"	Operator: transforms a ket into another: $\vert\chi\rangle = \hat{A}\,\vert\Psi\rangle$ They are called "operators"
8	Matrix element: the number $\mathbf{w}_n^\dagger\cdot A\cdot\mathbf{w}_m = A_{nm}$ is the matrix element A_{nm} of the matrix A.	Matrix element: the number $\int dx\, \varphi_n^*(x)\, \hat{A}\, \varphi_m(x) = A_{nm}$ is the matrix element A_{nm} of the operator $\hat{A}$.	Matrix element: the number $\langle n\vert\hat{A}\vert m\rangle = A_{nm}$ is the matrix element A_{nm} of operator A.
9	Eigenvalue problems: $A\cdot\mathbf{v} = a\,\mathbf{v}$	Eigenvalue problems: $\hat{A}\,\Psi(x) = a\,\Psi(x)$	Eigenvalue problems: $\hat{A}\,\vert\Psi\rangle = a\,\vert\Psi\rangle$
10	Explicit vector/matrix notation: $\mathbf{w}_2 = \begin{pmatrix} 0 \\ 1 \\ \vdots \end{pmatrix}$ $A = \begin{pmatrix} A_{11} & A_{12} & \cdots \\ A_{21} & A_{22} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$ $\mathbf{v} = \begin{pmatrix} c_1 \\ c_2 \\ \vdots \end{pmatrix}$ $\mathbf{v}^\dagger\bullet = (\,c_1^* \quad c_2^* \quad \cdots\,)\bullet$	Explicit vector/matrix notation: $\varphi_2 \triangleq \begin{pmatrix} 0 \\ 1 \\ \vdots \end{pmatrix}$ $\hat{A} \triangleq \begin{pmatrix} A_{11} & A_{12} & \cdots \\ A_{21} & A_{22} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$ $\Psi \triangleq \begin{pmatrix} c_1 \\ c_2 \\ \vdots \end{pmatrix}$ $\int dx \Psi^*(x)\_\_ = (\,c_1^* \quad c_2^* \quad \cdots\,)\bullet$	Explicit vector/matrix notation: $\vert 2\rangle \triangleq \begin{pmatrix} 0 \\ 1 \\ \vdots \end{pmatrix}$ $\hat{A} \triangleq \begin{pmatrix} A_{11} & A_{12} & \cdots \\ A_{21} & A_{22} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$ $\vert\Psi\rangle \triangleq \begin{pmatrix} c_1 \\ c_2 \\ \vdots \end{pmatrix}$ $\langle\Psi\vert = (\,c_1^* \quad c_2^* \quad \cdots\,)\bullet$