Homework H6 Solution

1. Turn in: An electron has mass $m_e \approx 9.1 \cdot 10^{-31}$ kg.

a. Its velocity has a range of 1 m/s. What is the range of positions that will be measured?

b. In a hydrogen atom, the range of velocities is closer to one million meters/second. (That electron is moving fast when it's squeezed into a small space!). What is the range of positions that will be measured in meters? In Ångstroms? In nanometers?

c. How does the length in b compare to what you know (remember freshman chem…) about the size of a 1s wavefunction?

Solution:

a. Starting with the Heisenberg uncertainty principle

$$\Delta x \Delta p = \frac{\hbar}{2} \tag{1}$$

we are given that Δv = 1m/s and so $\Delta p = m\Delta v = 9.1 \cdot 10^{-31}$ kg$\cdot$ m/s. From here, it is a simple matter of isolating Δx

$$\Delta x = \frac{\hbar}{2\Delta p} \tag{2}$$

followed by plug and chug. This gives $\Delta x = 5.79 \cdot 10^{-5}$ m.

b. We follow an exactly analogous procedure to that outlined in (a) above, only now we are given that $\Delta v = 1 \cdot 10^{6}$ m/s. The range of positions is thus $\Delta x = 5.79 \cdot 10^{-11}$ m = 0.579 Å = 0.0579 nm.

c. Recall that the most probable radius for an electron described by a 1s wavefunction is the bohr radius a_0 = 0.529 Å. This is very comparable with the results obtained in (b), differing by < 10%. The reason the 1s orbital in an H atom is about 0.5 Å in size is because the electrons really are moving with a velocity uncertain in the million m/s range!

2. Worked Problem 3.1 (Page 43). As a suggestion, try not reading the solution to the question immediately and see if you can work it out on your own. In any event, be sure you have looked this problem over; you will need to use the results in the following problem.

Solution: The full solution for this one is in the book. Read it over – could be exam material!

3. Problem 3.4 (Page 46)

Solution:

From the solution to Worked Problem 3.1 (found in the book) we know

$$\Delta\lambda\Delta x \geq \frac{\lambda^2}{4\pi} \tag{3}$$

so

$$\Delta x = \frac{\lambda^2}{4\pi\Delta\lambda} \tag{4}$$

The uncertainty in the wavelength is said to be one part in 10^7, so $\Delta\lambda = 1/10^7 = 10^{-7}$. For part (a), $\lambda = 0.5 \cdot 10^{-9}$ m. Plugging into (4) gives $\Delta x = 1.99 \cdot 10^{-13}$ m. For part (b), $\lambda = 5.00 \cdot 10^{-7}$ m. Plugging into (4) gives $\Delta x = 1.99 \cdot 10^{-7}$ m.

The TAKE-HOME message form this is that not just x and p are Fourier conjugate, but EVERYTHING else suffers variability in measurements also. Again, this variability is NOT an uncertainty, even though we sometimes use that language loosely. Uncertainty implies that you could do better. Quantum variability is intrinsic, just like a short enough sounds really does not have a precise pitch, no matter how much you measure its (nonexistent) pitch.