Homework H1 Solution

Problem 1: . Read the postulates...

Hope you read them! Who knows, Gruebele might do a pop quiz… just kidding!

Problem 2:

For a 3D particle, the kinetic energy

$$K = \frac{1}{2}mv^2 = \frac{(mv)^2}{2m} = \frac{p^2}{2m} = \frac{1}{2m}\left(p_x{}^2 + p_y{}^2 + p_z{}^2\right).$$

Thus it depends on all three momenta. Therefore,

$$H = K + V = \frac{p^2}{2m} + mgz = \frac{1}{2m}\left(p_x{}^2 + p_y{}^2 + p_z{}^2\right) + mgz$$

is the total energy (kinetic + potential).

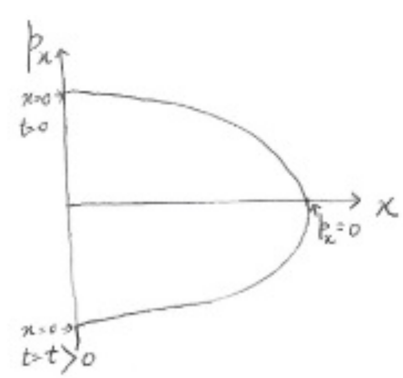
Thus, the kinetic energy depends on p_x and p_y also, even though the potential energy depends only on z. For example, if you throw a rock off a balcony, it flies away from the balcony sideways at constant velocity, and also drops downward.

If motion is restricted to the z axis only, $H = \frac{p_z{}^2}{2m} + mgz$. So, the Hamiltonian is a function of only two fundamental variables z and p_z.

Problem 3 (turned in, out of 0 to 2 points):

On an x-p_x plot (where x is now the vertical axis, not z), draw the trajectory of a marble that starts at *x*=0, is thrown straight up at t=0, then falls back into the hand at *x*=0 at *t*>0 later. I started the plot for you below.

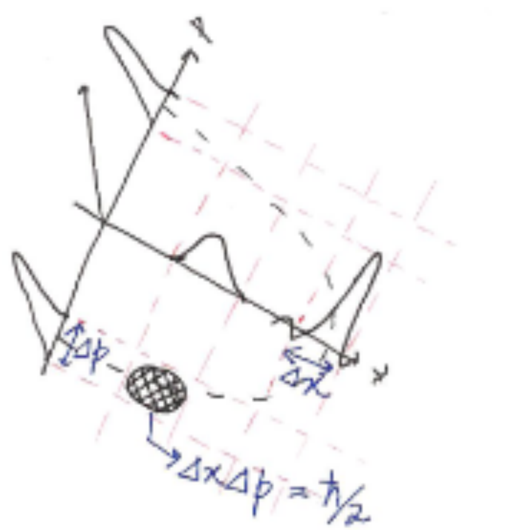
At t=0 (start of throw), x=0 and p_x is positive (considering velocity to be positive as the motion is along positive z axis). At the topmost part, there is zero velocity. As the marble comes down to the initial x=0 position, at t=t>0, its velocity (and momentum) is negative. Considering energy-momentum conservation, the final momentum is equal in magnitude and opposite in sign to the initial momentum.



Problem 4: Draw the quantum particle on the x-p plot for the same problem as in 3, but taking into account what Gruebele said about Δx and $\Delta p \neq 0$, i.e. the quantum particle's center of mass location is not a point on the x-p plot.

For the quantum particle, the 'trajectory' of the wavefunction will be similar, but keep in mind that position and momentum cannot be determined simultaneously! So, when we can determine x with accuracy, there'll be a finite uncertainty in p_x, and vice versa. The product of the two uncertainties, however, will remain constant ($\Delta x \Delta p_x = \frac{\hbar}{2}$).

In the picture below, you can see that the wavefunction Ψ(p) (as a function of momentum) starts at positive momentum, and goes to negative momentum. The wavefunction as a function of position (good old Ψ(x), the one you're more familiar with) starts out near x=0 (not shown), moves up to more positive x (2 examples shown) until it finally stops and turns around and comes back down. The center of the wavefunctions Ψ(x) and Ψ(p) traces out a path that looks pretty similar to the classical trajectory.



Next week, the TAs will tell you all about 'Fourier transforms', and the following week Gruebele will show you how x and p are related by the Heisenberg principle, and how the 'Fourier transform ' can be used to convert from Ψ(x) to Ψ(p). In physics, they love to use Ψ(p). In chemistry or Chem E applications, we usually just deal with Ψ(x).