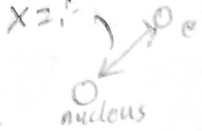
Lecture 7



- How big are molecules?

- $V(x) = \frac{-Ze^2}{4\pi\varepsilon_0 x}$

- $K = \frac{p^2}{2m_e}$ ← 9.1×10^{-31} kg (mass of electron)

- As particles move around, their K and V (kinetic & potential energy) exchange, while the total E is conserved.

$\Delta V \simeq \Delta K \approx \frac{E}{2}$ (Δ means range or variation)

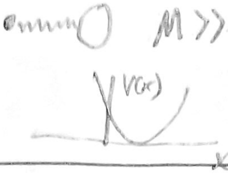
$|\Delta V| \approx \frac{-Ze^2}{4\pi\varepsilon_0 \Delta x} \simeq \Delta K \approx \frac{\Delta p^2}{2m_e}$ $\quad \Delta x \cdot \Delta p = \frac{\hbar}{2}$

$\Delta x \approx \frac{\pi\varepsilon_0\hbar^2}{2m_e Z^2 e^2} \approx 0.06$ Å for H atom

- In actuality, $\Delta x \approx 0.7$ Å. But this gets fairly close, and shows that atoms & molecules are small.

- Molecule as a spring

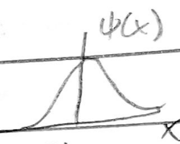
$M \gg m$ (for now, assume one mass is much higher.)



- We will see later that e⁻ pairs between nuclei act like springs holding nuclei together.
- $F = -kx$ → plug into Schrodinger
- $V(x) = \frac{1}{2}kx^2$

$$\left(\frac{-\hbar}{2m}\frac{d^2}{dx^2} + \frac{1}{2}kx^2\right)\psi_n(x) = E_n\psi_n(x)$$

- n subscript since more than 1 solution may be possible. In this case, there are countably infinite solutions.



- try $\psi_0 = e^{-ax^2}$
- $\frac{-\hbar}{2m}\frac{d^2}{dx^2}\psi_0 = \frac{-\hbar}{2m}\frac{d}{dx}\left(-2ax\,e^{-ax^2}\right) =$

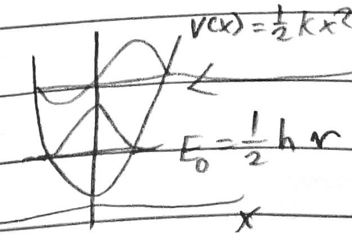
$$= \frac{\hbar^2}{2m}\left(2a - 4a^2x^2\right)e^{-ax^2}$$

$$\frac{\hbar^2}{2m}\left(2a - 4a^2x^2\right)e^{-ax^2} + \frac{1}{2}kx^2e^{-ax^2} = E_0\psi_0$$

- We want $\frac{\hbar^2}{2m}(-4a^2x^2) + \frac{1}{2}kx^2 = 0$

$\Rightarrow a = \frac{\sqrt{km}}{2\hbar}$ This cancels the x^2 terms.

- $E_0 = \frac{\hbar^2}{2m}2a = \frac{\hbar}{2}\sqrt{\frac{k}{m}} = \frac{h}{2}\nu \leftarrow \sqrt{\frac{k}{m}} = 2\pi\nu$



- You can repeat this analysis for higher n.