Lecture 6: Application of the Postulates

ex #1 (postulates 1 and 2):

- $\hat{H}\psi = \frac{-h}{2\pi i}\frac{\partial}{\partial t}\psi \rightarrow E = \frac{-h}{2\pi i}\frac{\partial}{\partial t}$

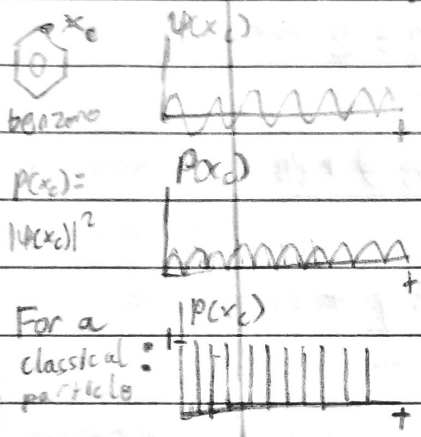
Energy and time: conjugate variables

$\Delta E \cdot \Delta t = \frac{h}{4\pi} = \frac{\hbar}{2}$

- $\nu = \frac{-1}{2\pi i}\frac{\partial}{\partial t}$ $\quad \Delta\nu \cdot \Delta t = \frac{1}{4\pi}$ (from music theory)
- $E = h\nu$ Planck's law
- For light, $c = \lambda\nu$. So all light has as specific wavelength, frequency, and thus energy.

ex #2 (postulate 3): (pg 5.)

- where's the electron (e^-)?



ex #3 (postulate 4) (pg 6).

- Can 2 e^- with identical spin be near each other.
- If near, $x_1 \approx x_2$
- For fermions (like e^-), $\psi(x_1, x_2) = -\psi(x_2, x_1)$
- For $x_1 = x_2$, $\psi(x_1, x_1) = -\psi(x_1, x_1)$
- Only possible if $\psi = 0$. $0 = -0$
- Probability of finding $2e^-$ with the same spin in the same place is $|\psi|^2 = 0$: Pauli Exclusion Principle: $\lim_{x_1 \to x_2} \psi(x_1, x_2) = 0$

ex 4: Energy (Postulates 1&2)

$H_{classical} = E = \frac{p^2}{2m} + V(x)$

In QM, $p = \frac{\hbar}{i}\frac{\partial}{\partial x}$ is not an independent variable.

In QM, $E = i\hbar\frac{\partial}{\partial t}$

$p^2 = p \cdot p = \left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)\left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right) = -\hbar^2\frac{\partial^2}{\partial x^2}$

$\hat{H}\psi = i\hbar\frac{\partial}{\partial t}\psi \rightarrow \left\{\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right\}\psi = E\psi$

$\downarrow$

$\hat{H}\psi = E\psi$

operator · eigenfunction · eigenvalue

ex.

$f = x^2$

$\hat{\Omega} = \frac{\partial}{\partial x}$

- f is not an eigenfunction of $\hat{\Omega}$, since $\hat{\Omega}f = \frac{\partial}{\partial x}(x^2) = 2x \neq cx^2 = cf$
- $f = e^{5x}$
- f is eigenfunction of $\hat{\Omega}$, since $\hat{\Omega}f = \frac{\partial}{\partial x}(e^{5x}) = 5e^{5x} = ce^{5x} = cf$