L6: review

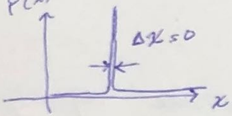
QM postulates

CM

① x, P , E, t } independent $\begin{cases} \Delta x \Delta p = 0 \\ \Delta E \Delta t = 0 \end{cases}$

② $-\dfrac{\partial V}{\partial x} = m \dfrac{\partial^2 x}{\partial t^2}$

③ $P(x)$



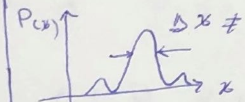
④ spin? Idk what you are talking about!

QM

$P = \dfrac{\hbar}{i}\dfrac{\partial}{\partial x}$, $E = -\dfrac{\hbar}{i}\dfrac{\partial}{\partial t}$ } conjugate $\begin{cases} \Delta x \Delta p = \dfrac{\hbar}{2} \\ \Delta E \Delta t = \dfrac{\hbar}{2} \end{cases}$

$\hat{H}\psi = i\hbar \dfrac{\partial \psi}{\partial t}$

$P(x) = |\psi(x)|^2 = \psi^*(x)\,\psi(x)$

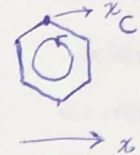


$$\text{Spin, } S = \begin{cases} \frac{1}{2}, \frac{3}{2}, \ldots & \psi(1,2) = -\psi(2,1) \\ 0, 1, 2, \ldots & \psi(1,2) = \psi(2,1) \end{cases}$$

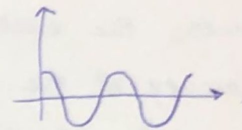
* electron and proton have $S = \frac{1}{2}$. photon has $S = 1$

today:

Page 5 exercise:

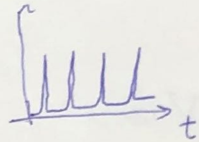


$\psi(x_c, t)$

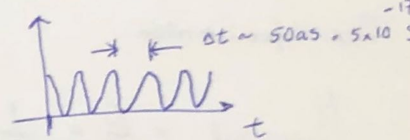


$P(x_c, t)$ $|\psi|^2$

$P_{cl}(x_c, t)$



CM



QM

P6 exercise: Pauli Exclusion Principle

e^- #1 $\vec{x}_1$ e^- #2 $\vec{x}_2$ } $\psi(\vec{x}_1, \vec{x}_2) = -\psi(\vec{x}_2, \vec{x}_1)$

what happens if I push them together?

$\vec{x}_1 = \vec{x}_2$; $\lim_{\vec{x}_1 \to \vec{x}_2} \psi(\vec{x}_1, \vec{x}_2) = \psi(\vec{x}_1, \vec{x}_1) = -\psi(x_1, x_2) \Rightarrow \psi \to 0$ as e^- #1 approaches e^- #2

* Pauli exclusion principle explains why the surface of a table blocks the passage of our hands; the electrons of table already occupy all the allowed spaces (i.e orbitals) and there is no more "room" for the electrons of our hand.

The Hamiltonian:

$$H_{classical} = E = \frac{1}{2m} p^2 + V(x)$$

In QM, we have:

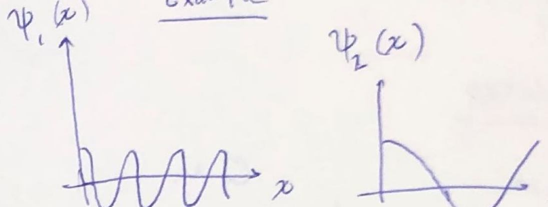
P2 : $\hat{H}\psi = i\hbar \frac{\partial \psi}{\partial t}$

P1 : $\underbrace{p = \frac{\hbar}{i}\frac{\partial}{\partial x}}$; $E = -\frac{\hbar}{i}\frac{\partial}{\partial t}$

$$p^2 = -\hbar \frac{\partial^2}{\partial x^2}$$

$$\boxed{\left\{-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right\}\psi = E\psi} \Rightarrow \hat{H}\psi = E\psi$$

Example



the "curvature" of $\psi_1(x) > \psi_2(x)$

$$\Rightarrow \frac{\partial^2}{\partial x^2}\psi_1(x) > \frac{\partial^2}{\partial x^2}\psi_2(x)$$

$$\Rightarrow P_1 > P_2$$

$\hat{H}\psi = E\psi$ is an eigenvalue equation

Homework Q 1.3 "order of things"

CM: $xp - px = 0$

QM: $\left\{x \frac{\hbar}{i}\frac{\partial}{\partial x} - \frac{\hbar}{i}\frac{\partial}{\partial x}\cdot x\right\} f \overset{(?=0)}{=}$

$$\frac{\hbar}{i}\left\{x\frac{\partial}{\partial x} - \frac{\partial}{\partial x}\cdot x\right\} f \neq 0$$

↓

apply the chain rule