Lecture 5

27 Jan 2023

Last time: Why QM?

① $E = \sum_i \frac{p_i^2}{2m} + V(x_i)$ kinetic + potential energy

ex: 

endothermic reaction

② independent variables a, b: $\Delta a \cdot \Delta b = 0$
conjugate variables, a, b: $\Delta a \cdot \Delta b > 0$

ex: Music $\Delta \nu \cdot \Delta t = \frac{1}{4\pi}$ {need long note duration to have well-defined pitch

ex: QM $\Delta x \cdot \Delta p = \frac{h}{4\pi}$ {$h \approx 6.22 \times 10^{-34}$ J·s not discovered until 20th century

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Today = the 4 postulates of QM

Some useful definitions:

① Hamiltonian: $H = E = \frac{p^2}{2m} + V(x)$

② Conjugate Variables: "derivative relationship"
$\Delta a \cdot \Delta b = \frac{c}{4\pi} \iff a = \frac{c}{2\pi i}\frac{\partial}{\partial b}$ or $b = \frac{-c}{2\pi i}\frac{\partial}{\partial a}$

③ Operator: Something that acts on a function

ex: function $= \psi$, operator = "5·" $\Rightarrow 5 \cdot \psi = 5\psi$

ex: function $= \psi$, operator $= \frac{\partial}{\partial x} \Rightarrow \frac{\partial}{\partial x}\psi = \frac{\partial \psi}{\partial x}$

In Class exercise:

Given time as a variable (t), then frequency (which is conjugate with t) is given by the operator $\hat{\nu} = \frac{1}{2\pi i}\frac{\partial}{\partial t}$

What does the operator $\hat{\nu}^2$ do to the function $\psi(t) = \sin(2\pi\nu_0 t)$

first: $\hat{\nu}^2 = -\frac{1}{4\pi^2}\frac{\partial^2}{\partial t^2} \Rightarrow \hat{\nu}^2\psi = -\frac{1}{4\pi^2}\left[\frac{\partial^2}{\partial t^2}\sin(2\pi\nu_0 t)\right]$

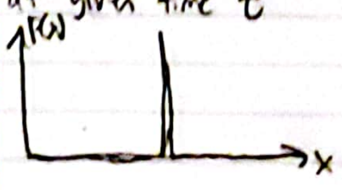
$$= -\frac{1}{4\pi^2}(2\pi\nu_0)\left[\frac{\partial}{\partial t}\cos(2\pi\nu_0 t)\right] = \underbrace{\nu_0^2}_{\text{eigenvalue}}\ \underbrace{\sin(2\pi\nu_0 t)}_{\text{eigenfunction}}$$

Goal of Quantum Mechanics

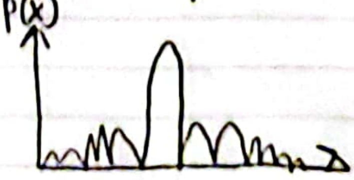
$\psi(x_1, x_2, \dots x_n, t)$ for e^- and the nuclei to find the probability of where these particles are

Postulates: Comparing CM to QM

CM	QM
① x, P are <u>independent</u> variables	① x, p are <u>conjugate</u> variables $x,\ \hat{p} = \frac{h}{2\pi i}\frac{\partial}{\partial x}$
② Equation of motion = $-\frac{\partial V}{\partial x} = F = ma = m\frac{\partial^2 x}{\partial t^2}$	② EOM: $\mathcal{H}\psi = i\frac{\partial}{\partial t}\psi$ note: $\frac{h}{2\pi} \equiv \hbar$ (used often) $\hat{E} = \frac{h}{2\pi i}\frac{\partial}{\partial t}$ "energy operator"
③ Probability of where particles are: Particle is exactly at position x at given time t	③ Prob. of where particle is: $P(x) = \lvert\psi(x)\rvert^2$



Ex.



CM

4 Concept does not exist

Spin = there are 2 kinds of particles:

Boson: $s = 0, 1, 2, \ldots$

Fermion: $s = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$

If I have 2 particles

$\psi_B(1,2) = +\psi_B(2,1)$

$\psi_f(1,2) = -\psi_f(2,1)$