TA Lecture Notes (08/31/22)

Review

1) Hamiltonian $H = E = \underbrace{\frac{p^2}{2m}}_{\text{kinetic energy term}} + \underbrace{V(x)}_{\text{Potential energy term}}$

2) The two (2) types of variables in nature

a) Independent (e.g., $x, y,$ and z)

b) Conjugate (e.g., $\underbrace{\Delta\nu}_{\text{Pitch}} \cdot \underbrace{\Delta t}_{\text{Duration}} = \frac{1}{4\pi}$ } in music)

Note! If a and b are conjugate variables with conjugate coefficient c

$\Delta a \cdot \Delta b = \frac{c}{4\pi}$, $b = \frac{c}{2\pi i}\frac{\partial}{\partial a}$, $a = \frac{-c}{4\pi i}\frac{\partial}{\partial b}$

here we see the inverse relationship between a and b

Today's Lecture

The Postulates of quantum mechanics (QM)

- Let's look at some definitions

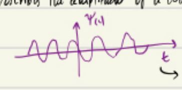
Operator: changes one function into another

example, let the function be $5x$, $(5x)y = 5y$ } multiplies y by 5 (for values of x)

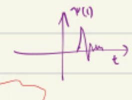
'$\frac{\partial}{\partial x}$', $\left(\frac{\partial}{\partial x}\right)x^2 = 2x$ } this operator takes the derivative of x

Wavefunction ($\Psi(x,t,\ldots)$): Describes the amplitude of a wave as a function of position (x) and time (t)

Example:



an example could be a flute tone



could be a snare drum

Exercise: $\Psi(t) = \sin(2\pi\nu \cdot t)$

Let's look at what happen when we apply the frequency operator $\hat{\nu}$

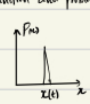
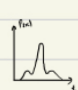
$\hat{\nu} = \frac{i}{2\pi}\left(\frac{\partial}{\partial t}\right)$, $\hat{\nu}\,\Psi = \frac{i}{2\pi}\left(\frac{\partial}{\partial t}\right)\sin(2\pi\nu t) = i \cdot \nu \cdot \cos(2\pi\nu t)$

and $\hat{\nu}^2\Psi = \frac{i}{2\pi}\left(\frac{\partial}{\partial t}\right)(i \cdot \nu \cdot \cos(2\pi\nu t)) = \nu^2 \sin(2\pi\nu \cdot t) = \nu^2 \cdot \Psi$

here you see that we get back the same function ($\sin(2\pi\nu \cdot t)$) and ν^2.

We call the ν^2 the eigenvalue and Ψ is called the eigenfunction

Now let's look at some postulates

	Classical Mechanics	Quantum Mechanics
1	Position (x) and momentum (p) are independent e.g. $\Delta x \cdot \Delta p = 0$	$p = \frac{h}{2\pi i}\left(\frac{\partial}{\partial x}\right)$ $\Delta x \cdot \Delta p = \frac{h}{4\pi}$ where $h = 6.62 \times 10^{-34}$ J·s
2	Equation of motion $\underbrace{-\frac{\partial V}{\partial x}}_{F} = m\underbrace{\frac{\partial^2}{\partial t^2}x}_{a}$, V = Potential, $F = ma$	$\hat{H}\Psi = i\hbar\frac{\partial}{\partial t}\Psi$, where $\hbar = \frac{h}{2\pi}$ Hamiltonian, often used interchangeably with energy, E
3	Wavefunction and probability  Probability is 'discrete'	$P(x) = \Psi(x)\Psi^*(x)$ $= \lvert\Psi(x)\rvert^2$,  } Probability might be 80% one area and 20% the next
4	N/A	Particles have weak magnetic fields, $s = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3$ and there are two (2) kinds of particles a) Bosons, $s = 0, 1, 2, 3, 4\ldots$ $\Psi(x_1, x_2) = \Psi(x_2, x_1)$ b) Fermions, $s = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$ $\Psi(x_1, x_2) = -\Psi(x_2, x_1)$