L5- review

classical mechanics: $E = \frac{p^2}{2m} + V(x)$

$F = -\frac{\partial V(x)}{\partial x}$, $\frac{\partial^2 x}{\partial t^2}$

x and p can be simultaneously specified

precisely: $\Delta p \cdot \Delta x = 0$

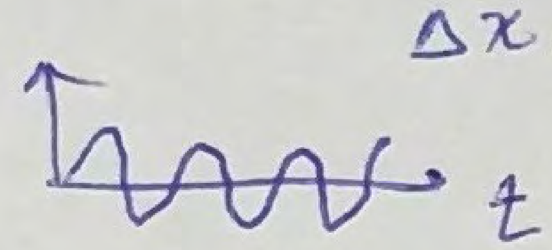
quantum mechanics:

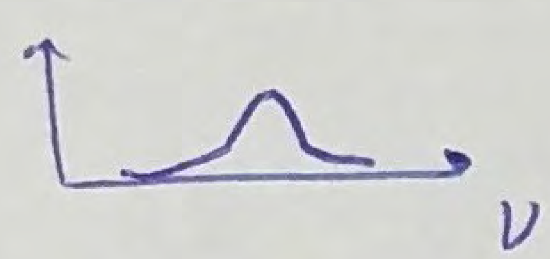
x and p are conjugate variable => they cannot be specified exactly simultaneously

$\Delta x \cdot \Delta p = \frac{h}{4\pi}$

Analogy to music

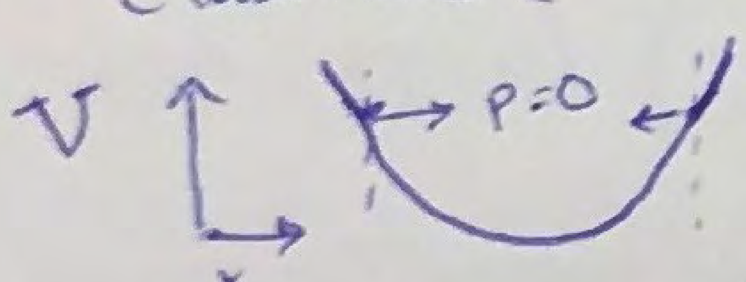
$\Delta \nu \cdot \Delta t = \frac{1}{4\pi}$



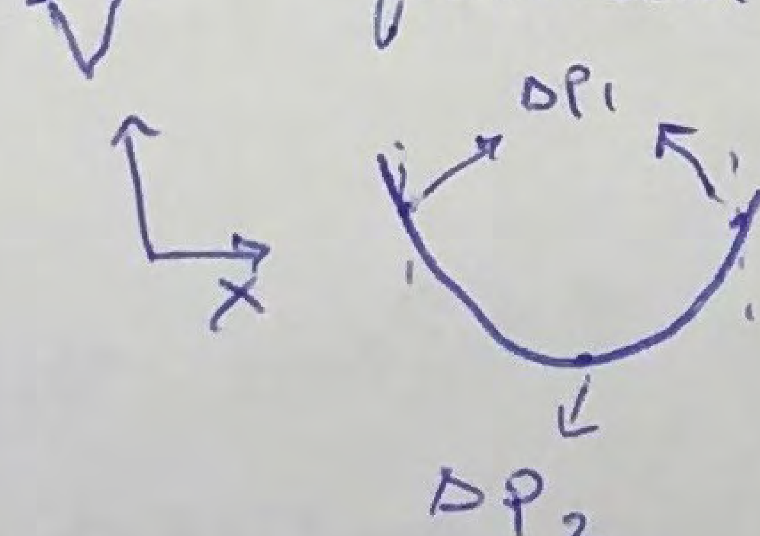


example: spring/harmonic potential:

classical



quantum



ΔP_1 and $\Delta P_2 > 0$

Today: postulates of QM

① Hamiltonian: $H = E = \frac{p^2}{2m} + V(x)$

② Conjugate variables, a and b:

$$b = \frac{c}{2\pi i}\frac{\partial}{\partial a} \; ; \; a = -\frac{c}{2\pi i}\frac{\partial}{\partial b}$$

Heisenberg principle $\Delta a \cdot \Delta b = \frac{c}{4\pi}$

martin: "Avoid calling this the Heisenberg uncertainty principle!"

③ Operators: Something that acts on a function. ex:

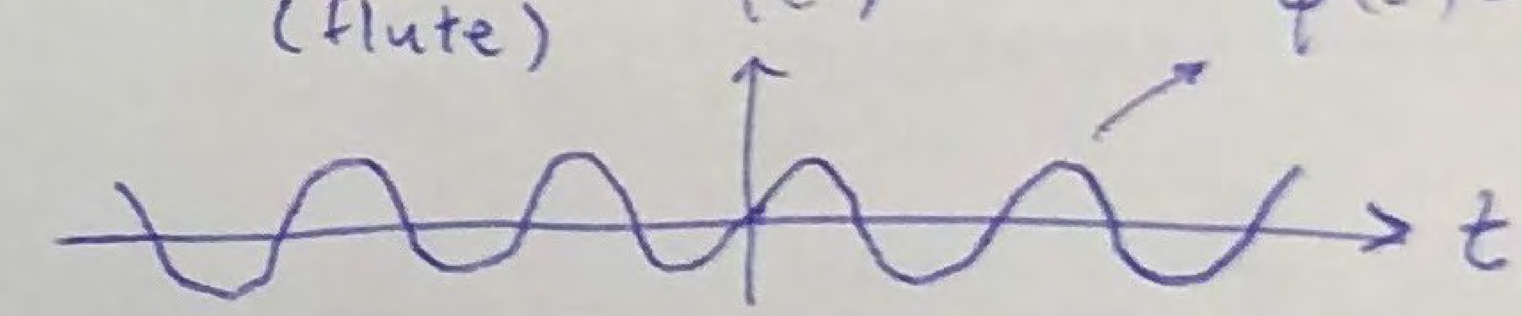
function = ψ ; 1 : $1 \cdot \psi = \psi$

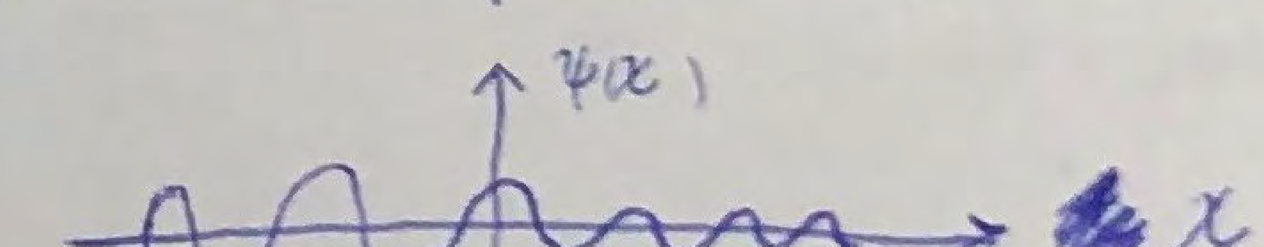
or

$\frac{\partial}{\partial x}$: $\frac{\partial}{\partial x}\psi = \psi'$

④ wave function: $\psi(x, t)$

ex: music (flute) $\psi(t)$ $\psi(t) = \sin(2\pi\nu t)$





exercise

(frequency operator) $\hat{\nu} = \frac{i}{2\pi} \frac{\partial}{\partial t} \Rightarrow \hat{\nu}^2 = \hat{\nu}.\hat{\nu} = \frac{-1}{4\pi^2} \frac{\partial^2}{\partial t^2}$

$\psi = \sin(2\pi \nu t)$; $\hat{\nu}^2 \psi = -\frac{1}{4\pi^2} \frac{\partial^2}{\partial t^2}[\sin(2\pi\nu t)]$

$= \nu^2 \sin(2\pi \nu t) = \nu^2 \psi$

Goals: System of e^- & nuclei with a wavefunction $\Psi(x_1, x_2 \cdots)$.

Find the probability of where the particles are.

(Note that ψ is not a function of p as p and x are not independent as per the Heisenberg principle)

Postulates:

1) x and p are conjugate with conjugation constant $c = h$

$\hat{p} = \frac{h}{2\pi i} \frac{\partial}{\partial x}$ $\quad \hat{x} = -\frac{h}{2\pi i} \frac{\partial}{\partial p}$

2) Equation of motion

"CM"

$-\frac{\partial V}{\partial x} = F = ma = m \frac{\partial^2 x}{\partial t^2}$

"QM"

$\hat{H}\psi = -i\hbar \frac{\psi}{\partial t}$

$\hbar = \frac{h}{2\pi}$

3) Meaning of wave function, ψ:

$P(x,t) = |\Psi(x,t)|^2 = \underbrace{\Psi^*(x,t)}_{\text{complex conjugate of } \Psi} . \Psi(x,t)$

$\Psi = a + ib$, $\Psi^* = a - ib$

$\Psi^* . \Psi = a^2 + b^2 = r^2$

Given an operator $\hat{A}$: $\bar{A} = \int dx\, \Psi^*(x,t) \hat{A} \Psi(x,t)$

Compare this with the average function from L1: $\bar{y} = \int dx\, y(x) P(x)$

4) Spin & Pauli exclusion:

Quantum particles have spin (S);

$S = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \ldots$

If I have a wave function for two particles at x_1, x_2

$$\psi(x_1, x_2) = -\psi(x_2, x_1) \text{ if } s = \tfrac{1}{2}, \tfrac{3}{2}, \tfrac{5}{2}, \dots$$

or $\psi(x_1, x_2) = +\psi(x_2, x_1)$ if $s = 0, 1, 2, \dots$