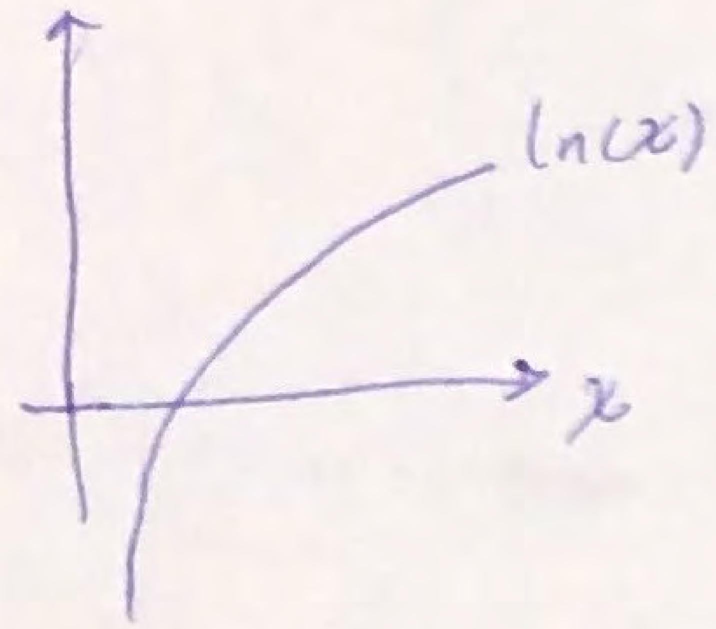
L4 - Review

math topics ...

⑤ $\ln(x)$



properties:

- monotonic $\Rightarrow$ unique
- compressed $\Rightarrow$ great for small and large numbers

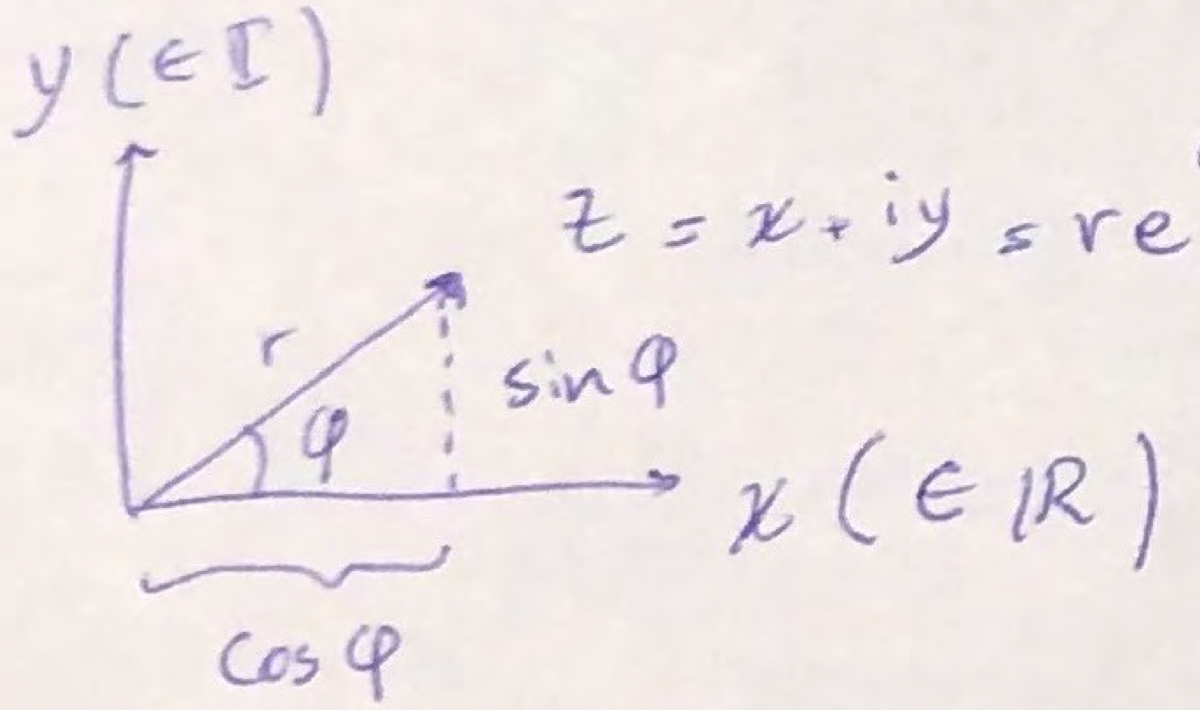
$\ln(a.b) = \ln(a) + \ln(b)$

turns multiplicative $\Longleftarrow$ into additive, e.g.

$$W_1 \cdot W_2 \xrightarrow{\ln} S_1 + S_2$$

⑥ complex numbers

$$e^{i\varphi} = \cos\varphi + i\sin\varphi \;;\quad i \equiv \sqrt{-1}$$



ex: for i, $r = 1$; $\varphi = 90^\circ$ (or $\frac{\pi}{2}$)

$$\left(i = 1 \cdot e^{i\frac{\pi}{2}} \right)$$

$1 \downarrow r$, $\;i\frac{\pi}{2} \downarrow \varphi$

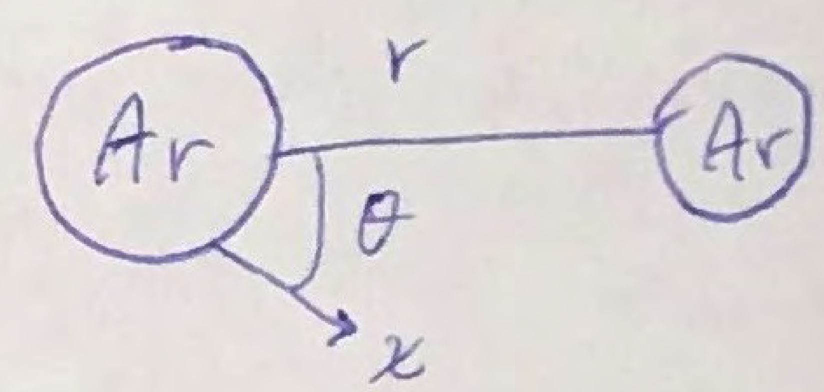
Today - Why go quantum?

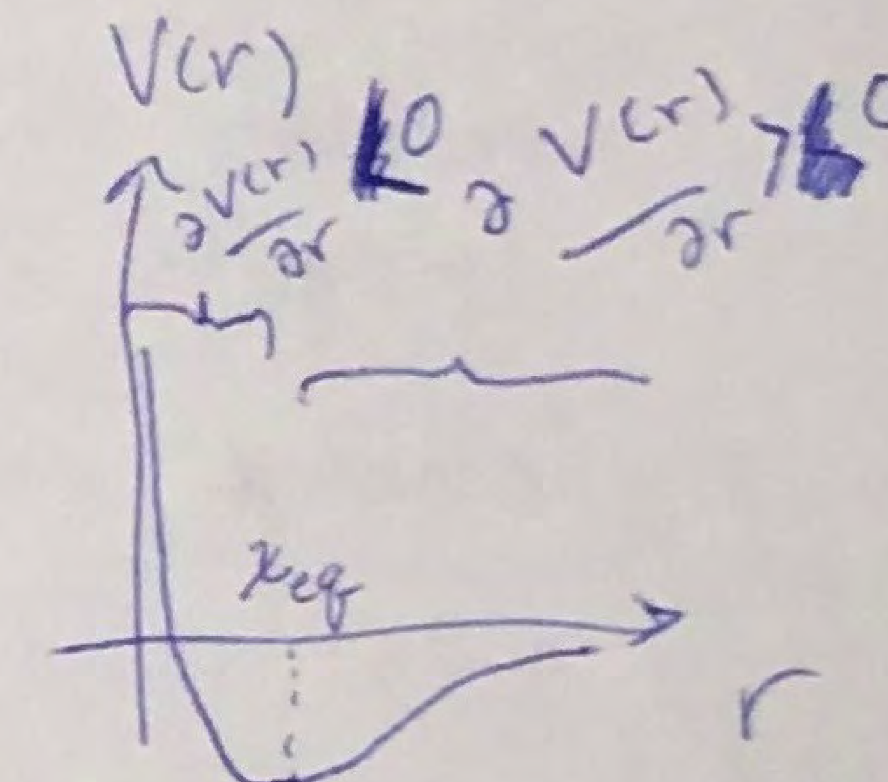
In classical mechanics, energy of particles =

$$E = \sum \frac{1}{2m_i} P_i^2 + V(x_i)$$

- $\sum \frac{1}{2m_i} P_i^2$: kinetic energy; P_i : momentum
- $V(x_i)$: potential energy
- i: just an index, not $i = e^{i\pi/2}$

ex:





$$F = -\frac{\partial V}{\partial r}$$

↙ force

$$F = ma = m\frac{\partial^2 x}{\partial t^2}$$

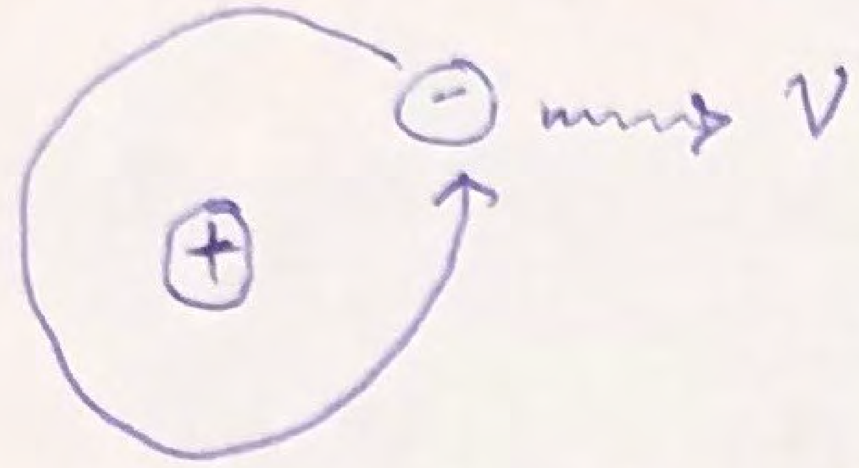
$$\Rightarrow -\frac{\partial V}{\partial r} = m\frac{\partial^2 x}{\partial t^2}$$

⇓

if $r > x_{eq}$ then the two atoms will attract each other.

If $r < x_{eq}$ the Ar atoms will repel each other.

Problems with classical mechanics :

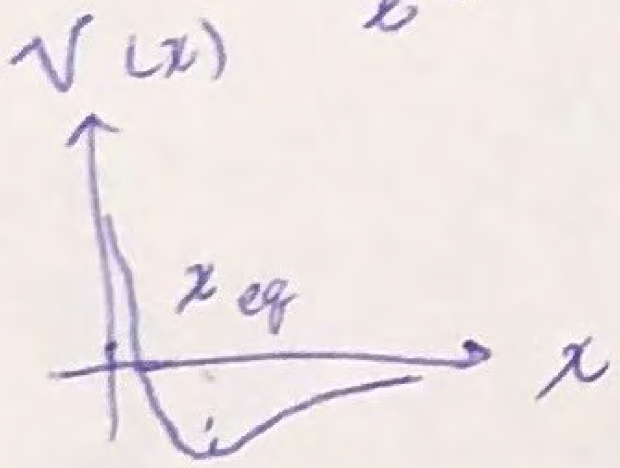
1) 

Classical mechanics : the negative charge is oscillating (the velocity is changing as it is orbiting) so according to electromagnetic laws it will emit light. Since light emission reduces the energy of the particle it gets closer to the positive charge and it will eventually collapse

⟹ Paradox !

2) where do you get $V(x)$ from?
(e.g. it is known empirically that

$$V \sim \frac{1}{x^6} \text{ for } x > x_{eq}$$



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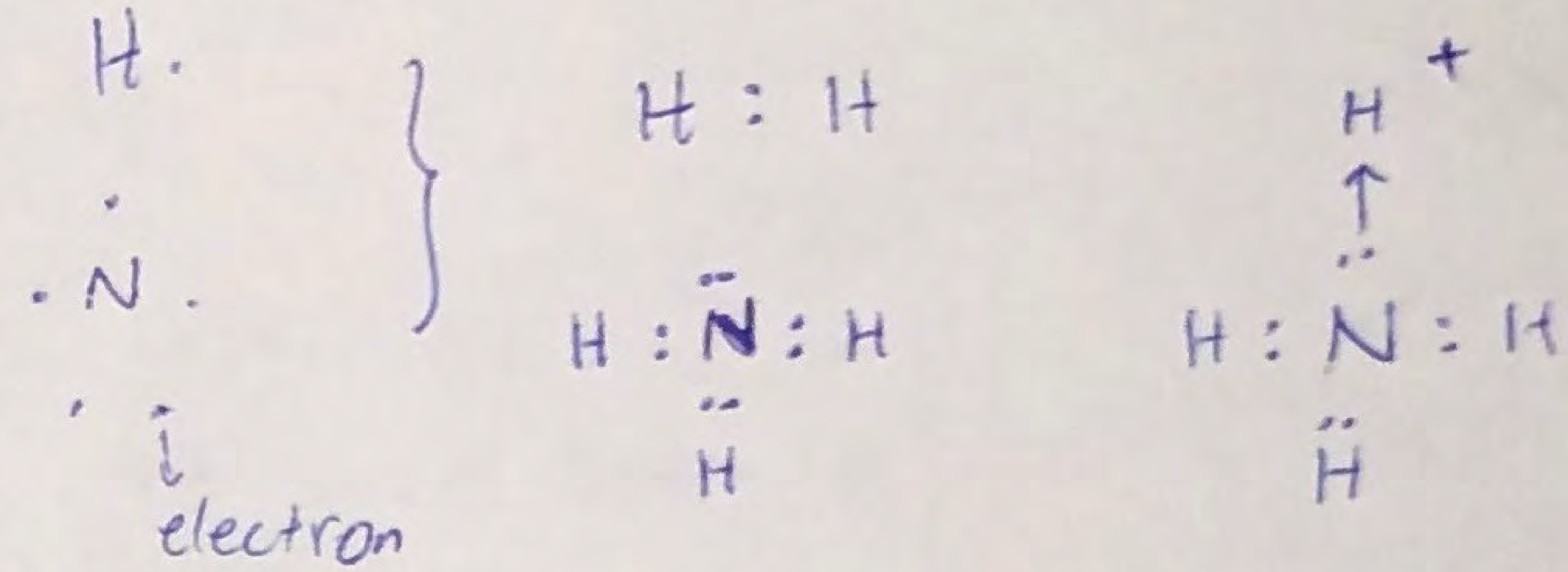
Early 1900s : GN Lewis :



Lewis's observation : in (almost) all cases the electrons get paired so the total number of electrons is even

Heisenberg : 2 kinds of variables exist:

1) independent : can be measured with arbitrary precision simultaneously

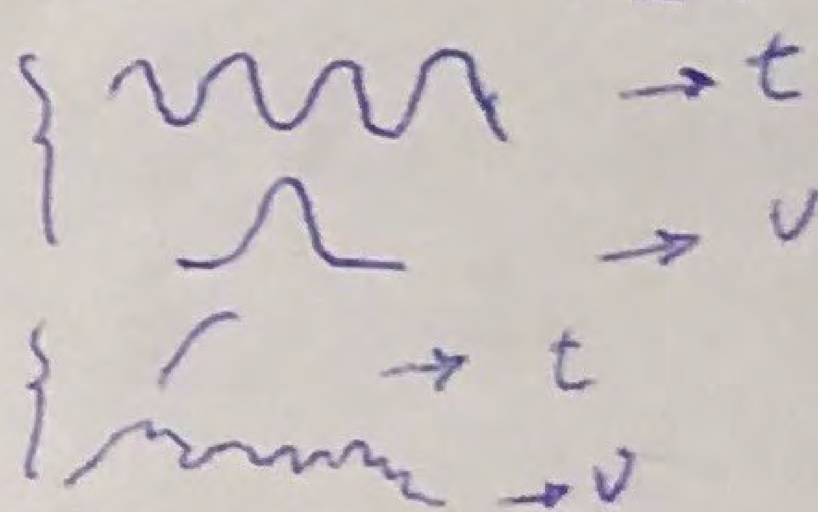
2) conjugate.

ex : music

duration (t) frequency (ν)

$$\Delta t \cdot \Delta \nu = \frac{1}{4\pi}$$; it was known that there is an uncertainty in ν and t

tones: guitar/flute ; drums



In classical mechanics : $\Delta x . \Delta P = 0$

(x and p can be measured precisely and simultaneously)

However, Heisenberg in 1925 ;

$\Delta x . \Delta P = \frac{h}{4\pi}$; $h \approx 6.62 \times 10^{-34}$ J.s

↓

plank constant

h is so small ⇒ it was not discovered until 1925.

Heisenberg's uncertainty has significance only in very small systems, e.g electrons