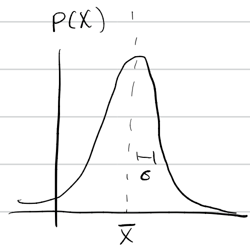
Lecture 3 8/25/23

Last Time:

③ Central Limit Thm

$X = \underbrace{X_1}_{\text{random variables}} + X_2 + \dots \Rightarrow P(X)$ is gaussian



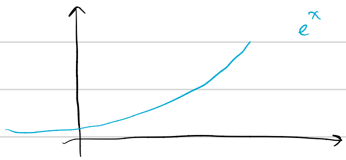
④ Bayes' Thm

$$\underbrace{P(A|B)}_{\text{probability that A happens given that B holds}} = P(B|A)\frac{P(A)}{P(B)}$$

Today: Final 2 math concepts

⑤ Logarithms $\begin{cases} \text{monotonic} \to \text{unique inverse} \\ \text{multiplication} \to \text{addition} \\ \text{large number} \to \text{small number} \end{cases}$

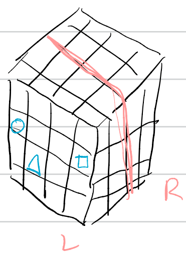
$\xrightarrow[\text{axes}]{\text{flip}}$



important: $\underbrace{z = xy}_{\text{multiply}} \Rightarrow \ln(z) = \underbrace{\ln(x) + \ln(y)}_{\text{add}}$

↳ extensive variables

Ex:



How many ways are there of putting the particles only on the left side?

$$W_{1/2} = 32 \cdot 31 \cdot 30 = 29760$$

ways of putting particles

How about the whole box?

$$W_{TOT} = 64 \cdot 63 \cdot 62 = 249984$$

Probability all particles on left side:

$$P(\text{all left}) = \frac{W_{1/2}}{W_{TOT}} \approx 0.12 \rightarrow 12\%$$

Now define $S = \ln W$

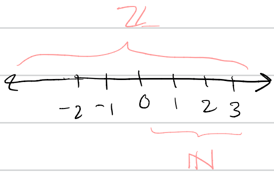
$$\ln\left(\frac{W_{1/2}}{W_{TOT}}\right) = \ln(W_{1/2}) - \ln(W_{TOT})$$

$$= S_{1/2} - S_{TOT}$$

$$= 10.3 - 12.4 = -2.1$$

$$S_{TOT} > S_{1/2}$$

6 Complex Numbers



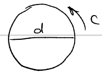
negative numbers

$c = g + g$

$\Rightarrow g = \frac{c}{2}$

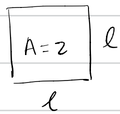
rational numbers

$\mathbb{Q}$



$c \sim d$

$\frac{c}{d} = \pi$



$l = \sqrt{2}$

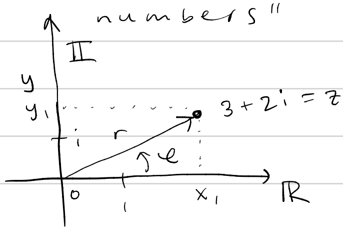
irrational numbers

$\mathbb{R} \setminus \mathbb{Q}$

Squareroot of any number (quadratic equations) (1600s).

$$\sqrt{-4} = \sqrt{4 \cdot -1} = 2\underbrace{\sqrt{-1}}_{= i}$$

To have the same rules, think of "complex numbers" as 2-dimensional numbers.

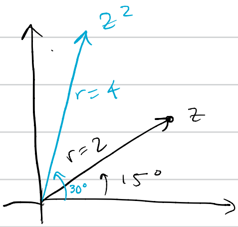


· Ordinary arithmatic

ex: $\left.\begin{aligned} z_1 &= x_1 + iy_1 \\ z_2 &= x_2 + iy_2 \end{aligned}\right\}$ $z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2)$

ex: $z_1 z_2 = x_1x_2 + x_1y_2 i + x_2y_1 i - y_1y_2$

$= (x_1x_2 - y_1y_2) + i(x_1y_2 + x_2y_1)$



Taylor series w/ complex numbers

$$y(x) = y(0) + \left.\frac{dy}{dx}\right|_{x=0} x + \frac{1}{2!} \left.\frac{d^2y}{dx^2}\right|_{x=0} x^2 + \dots$$

$$e^x = 1 + x + \frac{1}{2!} x^2 + \frac{1}{3!} x^3 + \dots$$

$$e^{i\varphi} = 1 + i\varphi - \frac{1}{2!} \varphi^2 - \frac{1}{3!} i\varphi^3 + \dots$$

$$= \underbrace{\left(1 - \frac{1}{2!} \varphi^2 + \dots\right)} + i\underbrace{\left(\varphi - \frac{1}{3!} \varphi^3 + \dots\right)}$$

$$= \cos\varphi + i \sin\varphi$$