Lecture 34 review! Two steps to transport: "particle flow" when μ not equal, i.e. $\frac{d\mu}{dx} \neq 0$

1. Find chemical potentials as a function of position, $\mu_i(x)$ for all species "i" in the system

2. Calculate flux $J_i = -u_i c_i \frac{d\mu_i}{dx}$ (Flux-molar force) ($\frac{\partial c}{\partial t} = -\frac{\partial J}{\partial x}$, continuity, or $J_i = 0$ for equilibrium)

ex: electrophoresis — ex: Ohm's Law — ex: Nernst Equation:

$v_{drift} = z \cdot \mathcal{E}$ ($z = \frac{\text{coulomb}}{\text{molar field}}$), $I = R^{-1} V_\theta$ ($R = \frac{L}{A}$ → series, parallel), $V_\theta = V_\theta^0 + \frac{RT}{q \cdot F} \ln c$ ($z e q \cdot F$)

Lecture 35: Some more transport/equilibrium examples

Osmosis:

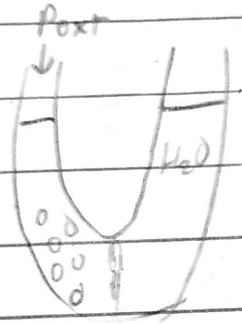
$\frac{\text{Liters}}{\text{mole}}$, $\frac{\partial G}{\partial P}$

1. $\mu_g = \mu^{(0)} + RT \ln C + v P$ (assuming 1 species)

2. $J = -u \cdot c \left(RT \frac{\partial \ln c}{\partial x} + v \frac{dP}{dx}\right)$

At equilibrium ($J = 0$)

$RT \frac{\ln C}{dx} + v \frac{dP}{dx} = 0$



$\rightarrow \frac{-v}{RT} \frac{dP}{dx} = \frac{\partial \ln C}{\partial x}$

$\frac{-v}{RT} \int_{x_1}^{x_2} dx' \frac{dP}{dx'} = \int_{x_1}^{x_2} dx' \frac{\partial \ln c}{\partial x'}$

$\frac{-v}{RT} \underbrace{(P(x_2) - P(x_1))}_{\Delta P} = \ln \underbrace{C(x_2)}_{C} - \ln \underbrace{C(x_1)}_{C_0}$

$\frac{-v}{RT} \Delta P = \ln \frac{C}{C_0} = \ln \frac{C_0 + \Delta C}{C_0} =$

$= \ln\left(1 + \frac{\Delta C}{C_0}\right) \approx \frac{\Delta C}{C_0}$

$\Delta P = \frac{-RT}{v} \cdot \frac{\Delta C}{C_0} = -\Delta C \cdot RT$

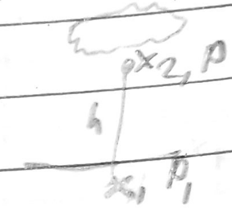
(v: $\frac{L}{mol}$, C_0: $\frac{mol}{L}$)

Gravitational equilibrium (g forces)

1. $\mu = \mu^{(0)} + RT \ln C + mgx$

2. $J = -u \cdot C \left(RT \frac{\partial \ln C}{\partial x} + mg \frac{\partial x}{\partial x}\right)$ 1

At equilibrium ($J = 0$)



$RT \frac{\partial \ln C}{\partial x} = -mg \frac{\partial}{\partial x} x$

$RT \int_{x_1}^{x_2} dx' \frac{\partial \ln C}{\partial x'} = -mg \int_{x_1}^{x_2} dx' \frac{dx'}{dx'}$

$RT(\ln C(x_2) - \ln C(x_1)) = -mg(x_2 - x_1)$

$RT \ln \frac{C_2}{C_1} = -mgh$

~~If we are talking about an ideal gas,~~

$\frac{C_2}{C_1} = \frac{P_2}{P_1} = \frac{P}{1\,atm}$

$RT \ln P = -mgh$

$P = e^{-\frac{mgh}{RT}} (1\,atm)$

- In a centrifuge, we make "g" artificially larger, so the necessary "h" is smaller to get a large concentration or pressure gradient