L35: review

2 steps to transport: Particles flow when

$$\mu_1 = \mu_2 \text{ or } \frac{\partial \mu}{\partial x} = 0$$

(1) Find $\mu_{ig}(x)$ for molecule "i"

in your system

(2) Calculate flux $J_i = -u_i C_i \dfrac{\partial \mu_{gi}}{\partial x}$

(flux ~ molar force)

example: electrophoresis

$$v_{drift} = z\,u\,\vec{E}$$

z ↓ charge, u ↓ mobility, $\vec{E}$ electric field

example: Ohm' law

$I = R^{-1} V_e$ ($R \sim \frac{L}{A}$ => series & parallel resistors)

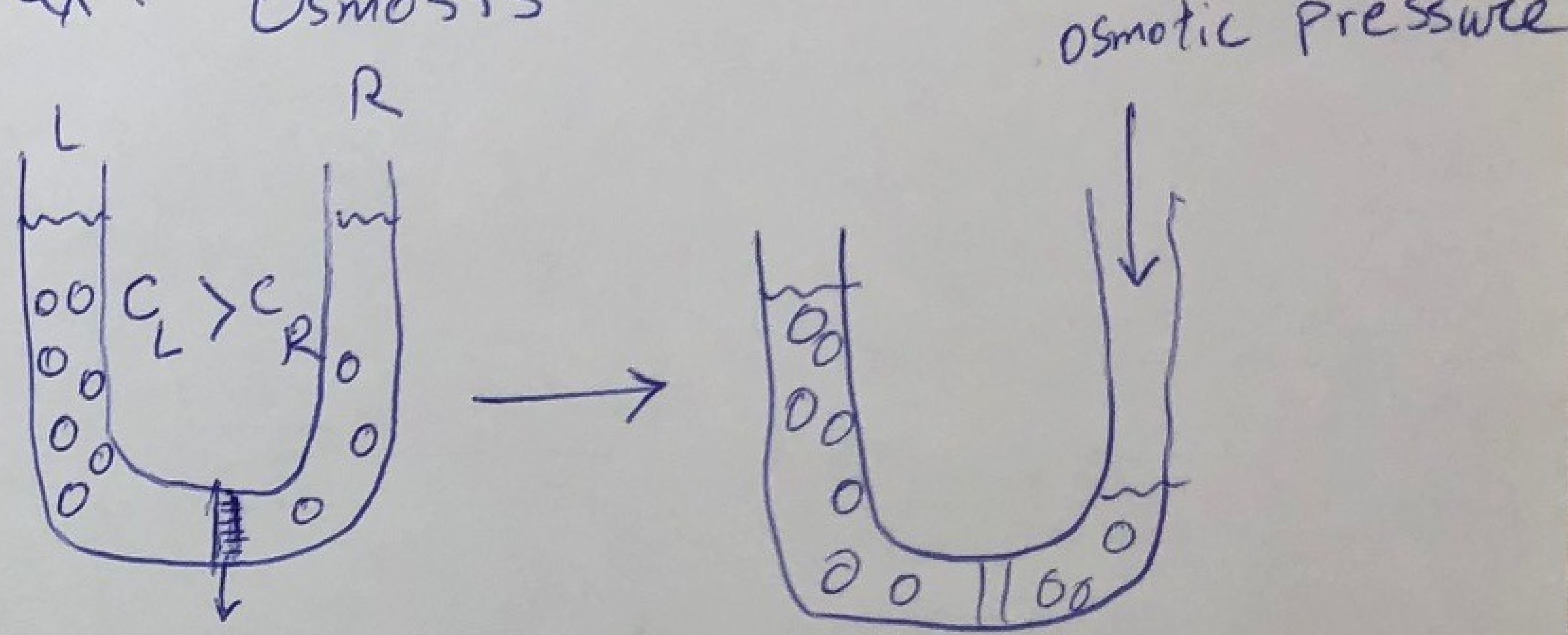
example: Nernst equation

$$V = V^0 - \frac{RT}{q_i F} \ln c_i$$

(add weights from stoichiometric coeff for full reaction

today: a couple of more examples

ex: Osmosis



membrane impermeable to particles (except solvent)

$C_L = C_R$

$\longrightarrow x$

* note that solvent moves to the more concentrated side so the chemical potential (or concentration) of the protein becomes equal.

$$\mu_g = \mu^0 + RT \ln c + \overset{\text{molar volume}}{v}\, P(x)$$

$$\frac{\partial \mu_g}{\partial x} = RT \frac{\partial \ln c}{\partial x} + v \frac{\partial P}{\partial x}$$

$$J = -uc\left(RT \frac{\partial \ln c}{\partial x} + v \frac{\partial P}{\partial x}\right)$$

Steady state: $J = 0$

$$\hookrightarrow RT \frac{\partial \ln c}{\partial x} = -v \frac{\partial P}{\partial x}$$

$$\Rightarrow RT \int_{x'=0}^{x'} dx' \frac{\partial \ln c}{\partial x'} = -v \int_{x'=0}^{x'} dx' \frac{\partial P}{\partial x}$$

$$\Rightarrow RT\left[\ln c(x) - \ln c(0)\right] = -v\left[P(x) - P(0)\right]$$

$$\Rightarrow -\frac{v\,\Delta P}{RT} = \ln \frac{c(x)}{c(0)} = \ln \frac{c(0)+\Delta c}{c(0)}$$

$$= \ln\left(1 + \frac{\Delta c}{c(0)}\right) \approx \frac{\Delta c}{c(0)}$$

In the ~~last~~ last line $\ln\left(1+\frac{\Delta c}{c(0)}\right)$ is approximated by the Taylor series:

$$\ln(1+x) = x + \underbrace{\cdots}_{\text{negligible}}$$

$$\Rightarrow \Delta P \approx -\frac{RT\,\Delta c}{v \cdot c(0)}$$

v: $\frac{\text{liter}}{\text{mole}}$, $c(0)$: $\frac{\text{moles}}{l}$

for ideal substance

$$\Delta P = -RT \underbrace{\Delta c}_{\frac{\Delta n}{V}}$$

example: barometric equation (gravitational equilibrium)

Assumption (ideal gas) : $P = \frac{n}{V} RT = c \underbrace{RT}_{\text{constant}}$

$$\Rightarrow \frac{P_2}{P_1} = \frac{c_2}{c_1}$$

$$\mu_g(x) = \mu^0 + RT \ln c + mgx$$

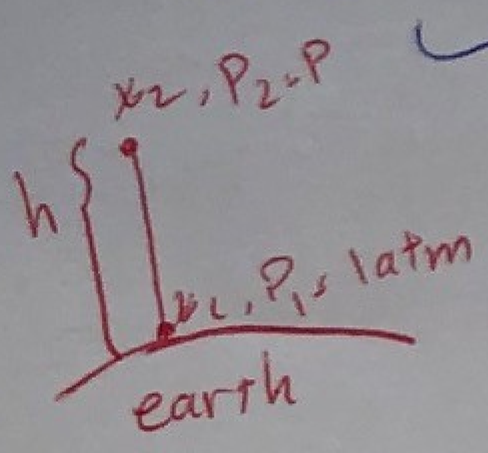
at equilibrium:

$$\frac{\partial \mu_g}{\partial x} = RT \frac{\partial \ln c}{\partial x} + mg = 0$$

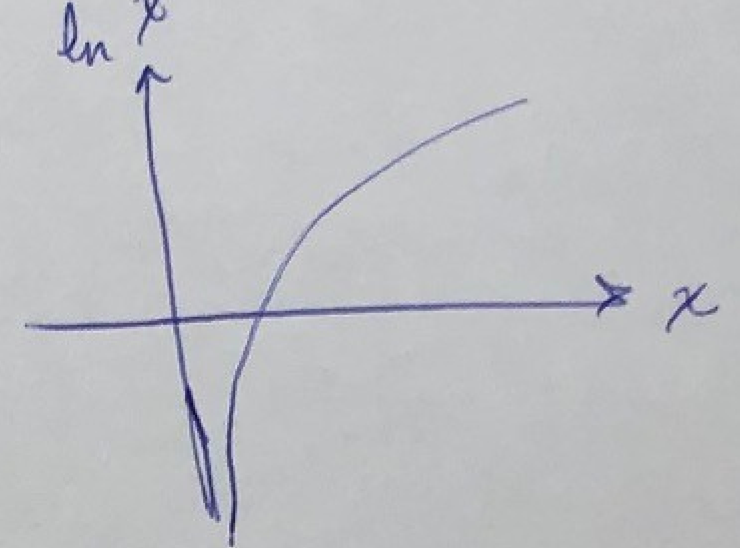
(If not at equilibrium : $\frac{\partial c}{\partial t} = -\frac{\partial c}{\partial t}$; $J = -uc\frac{\partial \mu}{\partial x}$)

$$\Rightarrow RT \int_{x_1}^{x_2} dx' \frac{\partial \ln c}{\partial x'} = -mg \int_{x_1}^{x_2} dx'$$

$$\Rightarrow RT \underbrace{\left[\ln c(x_2) - \ln c(x_1)\right]}_{\ln \frac{c_2}{c_1} = \ln \frac{P_2}{P_1}} = -mg(x_2 - x_1)$$



$$\Rightarrow RT \cdot \ln \overset{\to P_2/P_1}{P} = -mgh \Rightarrow \boxed{\ln P = -\frac{mgh}{RT}}$$



* note that we assumed : 1) T remains constant as h changes 2) g remains constant as h changes.

Table

	RT ln c	V_e	...	T
c				thermo phoresis
q	Nernst eq	ohm law	...	

$\to \mu_g = \mu^0 + RT\ln c + sT$

$\Downarrow$

$\frac{\partial \mu_g}{\partial x} = RT\frac{\partial \ln c}{\partial x} + s\frac{\partial T}{\partial x}$