Lecture 34

Last time: Transport Laws

(1) $\mu_{gi}(x)$ for System

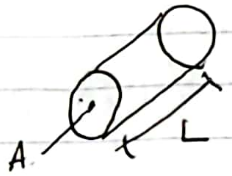
(2) $J = -U \cdot c \cdot \frac{d\mu_{gi}}{dx}$ $\Big\}$ ⇒ Law ex $\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2}$ Fick's Law

ex: $c(x) = c(0)\, e^{-\Delta\mu/RT}$ (μ_g^{tt}) 'Boltzmann' Law when $J = 0$

ex: $V_{drift} = \frac{\varepsilon \cdot z}{\gamma}$ (electric field; molar charge) (Protein gel) when $J \neq 0$

ex: $I = V/R$, $R = \frac{L\, RT}{z^2 \cdot D \cdot A \cdot c}$

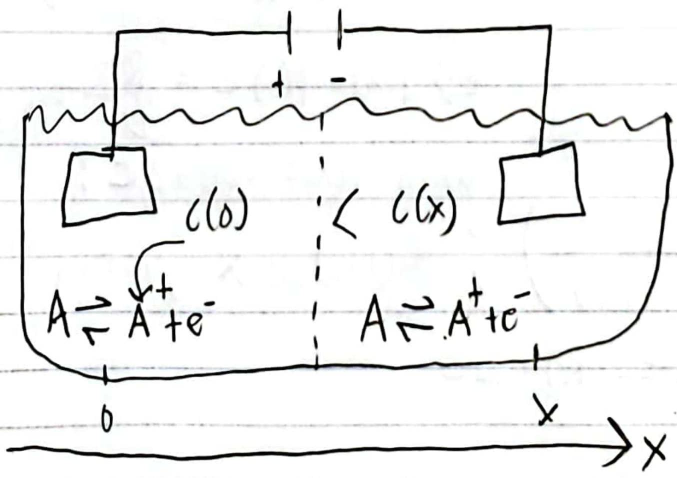
Length; concentration of charge carriers; Surface area; molar charge; diffusion coefficient



R: gas constant

today: More laws!

ex. Nernst equation; Simple case of ion "i" = "A" in solution at different concentrations



(1) $\mu_{gi}(x) = \mu_i(0) + RT \ln c_i + z_i \cdot V_e$

(2) $\frac{d\mu_{gi}}{dx} = RT \frac{d \ln c_i}{dx} + z_i \frac{dV_e}{dx}$

$$\Rightarrow \text{Flux of } A^+ = J = -U \cdot c \left(RT \frac{d\ln c}{dx} + \mathcal{F} \frac{dV_e}{dx}\right)$$

Voltage applied to stop $C_i(0)$ from equilizing with $C_i(x)$

Steady state: $$J = 0 \Rightarrow \frac{dV_e}{dx} = -\frac{RT}{z_i}\frac{d\ln c_i}{dx} = -\frac{RT}{q_i \mathcal{F}}\frac{d\ln c}{dx}$$

$q_i = \pm 1, \pm 2$

$\mathcal{F}$ = Faraday's constant

$$\int_0^x \frac{dV_e}{dx'}\,dx' = -\frac{RT}{q_i \mathcal{F}} \int_0^x \frac{d\ln c}{dx'}\,dx'$$

$$V_e(x) - V_e(0) = -\frac{RT}{q_i \mathcal{F}}\left(\ln C_i(x) - \ln C_i(0)\right)$$

$$\Rightarrow V_e(x) = V_e(0) - \frac{RT}{q_i \mathcal{F}} \ln\left(\frac{C_i(x)}{C_i(0)}\right)$$ <u>Nernst Equation</u>

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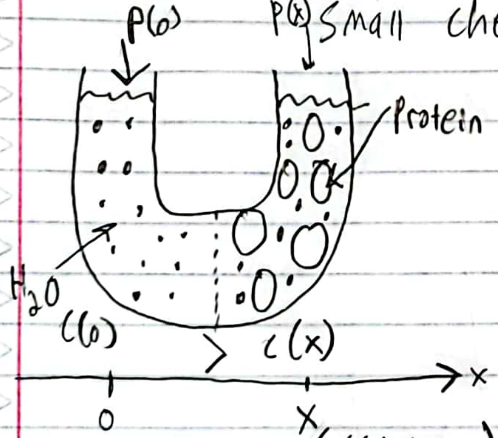
ex: Osmosis = Large chemical blocked from flowing, the small chemical <u>is not</u>.



$$J = -U \cdot c \cdot \left(RT \frac{d\ln c}{dx} + v \cdot \frac{dP}{dx}\right)$$ molar volume (v)

$$J = 0 \Rightarrow RT \frac{d\ln c}{dx} = -v \cdot \frac{dP}{dx}$$

$$\Rightarrow \underbrace{P(x) - P(0)}_{\Delta P} = -\frac{RT}{v} \ln \frac{c(x)}{c(0)}$$

with $c(x) = c(0) + \Delta c$:

$$\Delta P = -\frac{RT}{v} \cdot \ln\left(\frac{c(0) + \Delta c}{c(0)}\right)\ ; \quad \ln(1+x) \approx x\ , \ \therefore \ln\left(\frac{c(0)+\Delta c}{c(0)}\right) \approx \frac{\Delta c}{c(0)}$$

$v = \frac{1}{c(0)}$ $$\therefore \Delta P = \frac{-RT}{v\, c(0)} \cdot \Delta c = RT \cdot \Delta c$$

Appendix: