Lecture 33 review: Deriving transport equ.

1. Find $\mu_{gi}(x)$: $W(E) - S(E) \to E(S) \to G(T) \to \underbrace{\mu_i(T) = \frac{\partial G}{\partial n_i}}_{\mu_{gi}(T,x)}$

2a: Calculate flux: $(\text{mol}/\text{m}^2\cdot\text{s})$

$$J_i = c_i v_i = -U c_i \frac{\partial \mu_{gi}}{\partial x} \quad \left(\gamma = \frac{1}{U} = \frac{RT}{D}\right)$$

2b: Use continuity equation:

$$\frac{\partial c_i}{\partial t} = \frac{-\partial J_i}{\partial x} \quad (\text{in 1D})$$

ex:

1. $\mu_{gi} = \mu^{(0)} + RT\ln[c(x)]$ (Ideal solution, 1 solute)

2a: $J = -URT\frac{\partial c}{\partial x} = -D\frac{\partial c}{\partial x}$

2b: $\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2}$ (Fick's Law)

Lecture 34: Laws of the day:

ex: Faraday's law of electrophoresis:

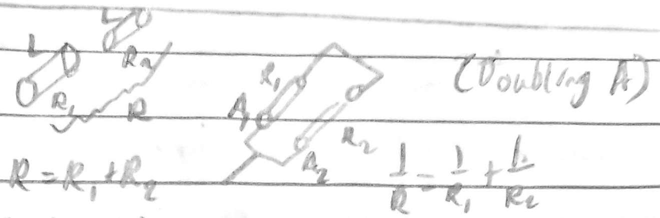
system: $\mu^{(0)}$ is constant, molecules have molar charge z in potential $V_e = -x\cdot\mathcal{E}$

1. $\mu_g = \mu^{(0)} + zV_e$ $\qquad \frac{\partial z}{\partial x}V_e = 0$, since molar charge is constant

2. $\frac{d\mu_g}{dx} = 0 + z\frac{\partial V_e}{\partial x} = z\cdot -\mathcal{E} = -z\mathcal{E}$

$$J = -Uc\frac{d\mu_g}{dx} = c\underbrace{Uz\mathcal{E}}_{v} = cv$$

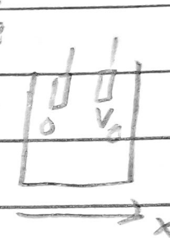
$$v_{drift} = U_z\mathcal{E} = \frac{-z}{\gamma}\mathcal{E} \quad (\text{Faraday's law})$$



$R = R_1 + R_2$ (Doubling L) $\qquad \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ (Doubling A)

ex: Nernst Equation (concentration form)

1. $\mu_{gi} = \mu_i^{(0)} + RT\ln c_i(x) + z_i V_e$



2. $\frac{d\mu_{gi}}{dx} = 0 + RT\frac{\partial \ln c_i}{\partial x} + z_i\frac{\partial V_e}{\partial x}$

$$J = -U_i c_i\left(RT\frac{\partial \ln c_i}{\partial x} + z_i\frac{\partial V_e}{\partial x}\right)$$

$J = 0$ at equilibrium

$$\to \frac{\partial V_e}{\partial x} = \frac{-RT}{z_i}\frac{\partial \ln c_i}{\partial x} \quad \left(\text{integrate from 0 to x on both sides}\right)$$

$$\to \int_0^x dx'\frac{\partial V_e}{\partial x'} = \frac{-RT}{z_i}\int_0^x \frac{d\ln c_i}{dx'}dx'$$

$$V_e(x) - V_e(0) = \frac{-RT}{z_i}\left(\ln c_i(x) - \ln c_i(0)\right)$$

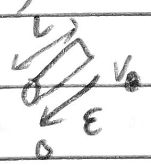
$$V_e(x) = V_e(0) - \frac{RT}{n_i F}\ln\frac{c_i(x)}{c_i(0)}$$

n_i: Integer charge; F: Faraday's Constant

ex: Ohm's Law:

$$\text{Current}\left(\frac{C}{s}\right) \propto \text{Flux}\left(\frac{\text{moles}}{m^2\cdot s}\right)$$

$$I = z\left(\frac{C}{\text{mol}}\right)\cdot J\left(\frac{\text{moles}}{m^2\cdot s}\right)\cdot A(m^2)$$



$$= z\cdot cv\cdot A$$

$$= z\cdot cUz\mathcal{E}\cdot A$$

$$= z^2cU\frac{V_e}{L}\cdot A$$

$$I = \frac{V_e}{R}, \quad R = \frac{L}{z^2cUA} \Leftrightarrow IR = V_e \;(\text{Ohm's law})$$

$$R = \frac{LRT}{z^2cDA} \quad \begin{cases} \uparrow \text{ with } T \\ T\to 0,\ R\to 0 \text{ (superconductor)} \end{cases}$$

$$\frac{Lk_BAT}{z^2cDA_{rea}}$$