Lecture 33

Last Time: Physico-chemical mechanics

Molecular motion equalizes $\mu(x_1)$ and $\mu(x_2)$ but how fast?

$D/RT = 1/\gamma$

P1: $\mu_{gi} = \mu(\dots, x)$ $\quad P_2 = J = -u\,c\,\dfrac{d\mu_{gi}}{dx} = v\,c$

Always true from math: $\dfrac{\partial J}{\partial x} = -\dfrac{\partial c}{\partial t}$

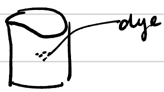
How to derive transport laws:

1) Find $\mu_g(x)$ for your system.

2) Plug into postulate 2: $J \neq 0$ out of equilibrium

$J = 0$ in equilibrium or steady state

ex:

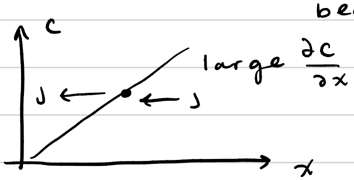


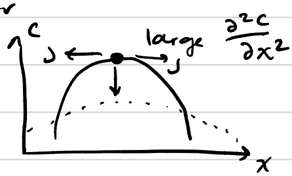
$\mu_g(x) = \mu^{(0)} + RT \ln c(x) \Rightarrow \dfrac{\partial c}{\partial t} = D\dfrac{\partial^2 c}{\partial x^2}$

does not depend on position in beaker

Fick's law

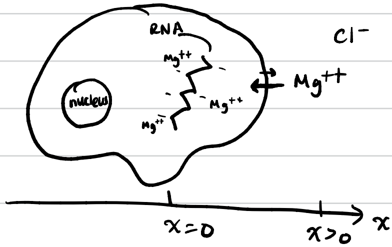
So,





Today: "Any law you want," part 1

ex: steady state of Mg^{++} in a cell



Mg^{++} binds to RNA and since $[RNA]_{cell} > [RNA]_{outside}$ the Mg^{++} conc will not be in equilibrium, but $J = 0$ when it reaches steady state inside and outside of the cell.

our system: $\mu_g(x) = \mu^{(0)}(x) + RT \ln c(x)$

② $\Rightarrow$ $J \sim \dfrac{d\mu_g}{dx} = \dfrac{\partial \mu^{(0)}}{\partial x} + RT \dfrac{\partial \ln c(x)}{\partial x} = 0$

$\Rightarrow d\mu^{(0)} = -RT\, d(\ln c)$

$\Rightarrow \displaystyle\int_{\mu^{(0)}(x=0)}^{\mu^{(0)}(x)} d\mu^{(0)} = -RT \int_{c(x=0)}^{c(x)} d(\ln c)$

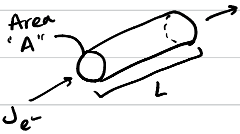
$\Delta\mu^{(0)} = \mu^{(0)}(x) - \mu^{(0)}(0) = -RT\left(\ln(c(x)) - \ln(c(0))\right)$

$\Delta\mu^{(0)} = -RT \ln\left(\dfrac{c(x)}{c(0)}\right)$

$\Rightarrow e^{-\Delta\mu^{(0)}/RT} = c(x) / c(0)$

$\Rightarrow c(x) = c(0)\, e^{-\underbrace{\Delta\mu^{(0)}}_{\text{due to RNA binding}}/RT}$

ex: Ohm's law



$\left(\dfrac{\text{moles}}{m^2\, s}\right)$

$\rightarrow I$ = current (amperes = $\dfrac{\text{coulombs}}{s}$)

molar charge ~96000 for electrons

$I\left(\dfrac{\text{coulombs}}{s}\right) = \underbrace{J\left(\dfrac{\text{moles}}{m^2\, s}\right)}_{= v_{drift} \cdot c} \cdot A\,(m_2) \cdot z\left(\dfrac{\text{coulombs}}{\text{mol}}\right)$

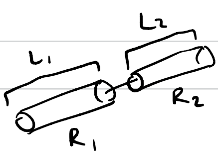
$= v_{drift} \cdot c \cdot A \cdot z$

proved (for a protein on a gel): $v_{drift} = z \mathcal{E} U = \dfrac{z\mathcal{E}}{\gamma}$ electric field $\mathcal{E} = \dfrac{V_e}{L}$

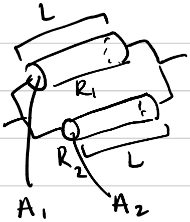
$\Rightarrow$ combine $I = \dfrac{z U V_e}{L} \cdot A \cdot z$

$I = \dfrac{V_e}{R}$ w/ R "resistance" $= \dfrac{L\,RT}{z^2 D \cdot c \cdot A}$ (R = gas constant)

"ohm's law"



$\Rightarrow R = \dfrac{(L_1 + L_2)RT}{z^2 DcA} = \dfrac{L_1 RT}{z^2 Dca} + \dfrac{L_2 RT}{z Dca} = R_1 + R_2$



$$\frac{1}{R} = \frac{z^2 DCA}{LRT} = \frac{z^2 Dc A_1}{LRT} + \frac{z^2 DcA_2}{LRT}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$