Lecture 32 | review!

From the Langevin equation:

$F_{appl} + F_{friction} + F_{random} = ma \approx 0$

$\uparrow$ $-\frac{\partial \mu}{\partial x}$ $\quad$ $\uparrow$ $-\gamma v$

• We can derive P.C.M.

$\boxed{\mu(x_1) > \mu(x_2)}$ $\xrightarrow{\text{flux } J \neq 0}$

" μ is equalized when particles stop flowing "

$\left(\frac{\text{Joules}}{\text{mole}\cdot m} = \frac{N}{mol}\right)$ force per mole of particle

$-\frac{\partial \mu}{\partial x} = F_{appl}$

or " difference in μ is driving force for particle flow "

P1: $\mu_i(T, P, \ldots) \rightarrow \mu_{gi}(T, P, \ldots, x)$ chemical potential can depend on x

P2: $J_{xi} = c\, v_{drift} = \frac{c}{\gamma} F_{appl} = U_c \frac{\partial \mu_{gi}}{\partial x}$

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ex #3: Fick's Law of Diffusion

How does $\frac{\partial c}{\partial t}$ depend on gradient $\frac{\partial c}{\partial x}$?

• Assume only 1 chemical (drop " i ")

• " chemical potential $\mu^{(0)}$ = constant

• " Temperature, pressure, etc., are constant

• $\mu = \mu^{(0)} + RT \ln c$

$\frac{d\mu}{dx} = \frac{d\mu^{(0)}}{dx} + RT \frac{d \ln c}{dx} = 0 + RT \frac{\ln c}{dx}$

• ~~use postulate to write down flux:~~

~~$J_x = -RT \mu c \frac{d \ln c}{dx}$~~ $\left(J_x = U_c \frac{\partial \mu_c}{\partial x}\right)$

$= -RT \mu \frac{dc}{dx}$ (Fick's 1st Law)

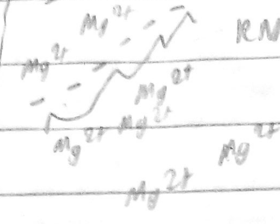
Continuity eq: $\frac{\partial c}{\partial t} = \frac{-\partial J}{\partial x}$

$\frac{\partial c}{\partial t} = \frac{-\partial J}{\partial x} = RT\mu \frac{\partial^2 c}{\partial x^2}$

$\uparrow$ $\frac{RT}{\gamma}$ $\uparrow$

$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2}$ (Fick's 2nd Law)

ex #2: HWK II.2



$\mu^{0}_{Mg^{2+}}$ (near RNA)

$\wedge$

$\mu^{0}_{Mg^{2+}}$ (far from RNA)

Lecture 33: (read right to left)

Let's calculate some particle laws:

ex: #1: equilibrium

$J = \mu_i \frac{\partial \mu_{gi}}{\partial x}$

$\mu_{gi} = \mu_i^{(0)} + RT \ln(c_i)$

$J = 0$ at equilibrium

$\rightarrow \frac{\partial \mu_{gi}}{\partial x} = 0$

$0 = \frac{d\mu_i^{(0)}}{dx} + RT \frac{d \ln(c_i)}{dx}$

$d\mu_i^{(0)} = -RT\, d\ln(c_i)$

$\int_0^{\mu_i(x)} d\mu_i^{(0)} = -RT \int_{c_0}^{c_i(x)} d\ln(c_i)$

$\mu_i^{0}(x) = -RT\left(\ln c_i(x) - \ln c_0\right)$

Apply $e^{\wedge}$ to both sides

$c_i(x) = c_i(0)\, e^{-\mu_i^{(0)}(x)/RT}$

Useful formulas:

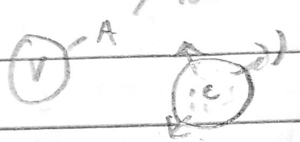
$\frac{\partial^2 G}{\partial n_i \partial T} = \frac{\partial^2 G}{\partial T \partial n_i}$

$\frac{\partial}{\partial n_i}\frac{\partial G}{\partial T} = \frac{\partial}{\partial T}\frac{\partial G}{\partial n_i}$

$\frac{\partial}{\partial n_i}(S) = \frac{\partial}{\partial T}\mu_i$

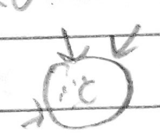
$-S_i = \frac{\partial \mu_i}{\partial T}$

~~Continuity formula:~~



$D \cdot J = \frac{\partial J}{\partial x} + \frac{\partial J}{\partial y} + \frac{\partial J}{\partial z} > 0$

$\frac{\partial c}{\partial t} < 0$



$D \cdot J < 0$ $\quad \frac{\partial c}{\partial t} > 0$

$D \cdot J = 0$ $\quad \frac{\partial c}{\partial t} = 0$

$\frac{\partial c}{\partial t} = \frac{-\partial J}{\partial x}$

"continuity equation"