

Last Time: Transport

Isolated system:  $F = ma$

Open system:  $F = F_{\text{applied}} + F_{\text{random}} + F_{\text{friction}} = 0$

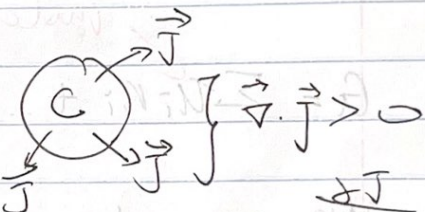
$$\Rightarrow -\frac{du}{dx} + F_{\text{random}} - \delta v = 0$$

$\Rightarrow -\frac{du}{dx} = \delta v$  if we neglect the random force

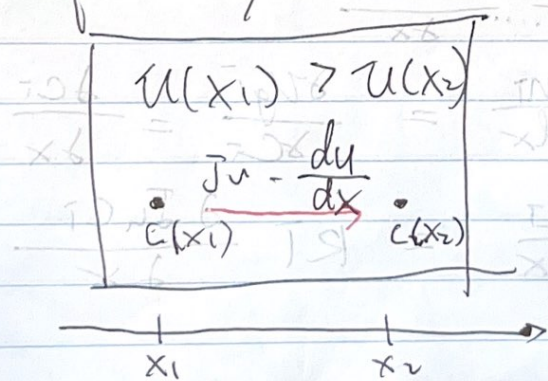
ex:  $v_{\text{drift}} = \frac{E \cdot Q}{\delta}$  for protein on gel.

Flux:  $J \left( \frac{\text{mole}}{\text{m}^2 \cdot \text{s}} \right) = v \cdot c = -\frac{1}{\delta} \frac{du}{dx} \cdot c = -U \cdot c \cdot \frac{du}{dx}$

Also: p. 531 always true

$-\frac{dI}{dx} = \frac{dc}{dt}$ , reason:   $\int \vec{j} \cdot \vec{j}' > 0$  or  $\frac{dI}{dx} > 0$

Today: Postulate of physicochemical mechanics, or "How do molecule move in an open system?"



Flux moves particle to lower chemical potential.

For ideal solutions,  $\mu = \mu^{(0)} + RT \ln C$

$\Rightarrow$   $C(x_2)$  increase until  $\mu(x_1) = \mu(x_2)$   
 $C(x_1)$  decrease. e.g.  $-3.2 \cdot x$

P1:  $\mu_i(T, P, n_i, \dots) \rightarrow \mu_{gi}(T, P, n_i, \dots, x)$   
 Assumed: we only talk about volume & V large enough so  $N \gg \bar{N}$ .

P2:  $\bar{J}_x i = - \mu_i C_i \frac{d\mu_{gi}}{dx}$   
 $\leftarrow$  "Flux"  $\leftarrow$  "molar force"

P1  $\Rightarrow \frac{d\mu_{gi}}{dx} = \frac{\partial \mu_{gi}}{\partial x} + \frac{\partial \mu_{gi}}{\partial T} \frac{dT}{dx} + \frac{\partial \mu_{gi}}{\partial P} \frac{dP}{dx} + \frac{\partial \mu_{gi}}{\partial n_i} \frac{dn_i}{dx}$   
 $\leftarrow$  "molar free energy"

$G = \sum \mu_i n_i + \dots$

We can also simplify the 1<sup>st</sup> & last term in this eq.

Assume an ideal system:

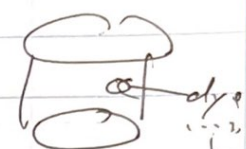
$\mu_{gi} = \mu_i^{(0)} + RT \ln C_i$

$\Rightarrow \frac{\partial \mu_{gi}}{\partial x} = \frac{\partial \mu_i^{(0)}}{\partial x}$

$\Rightarrow \frac{\partial \mu_{gi}}{\partial n_i} \frac{dn_i}{dx} = \frac{\partial \mu_{gi}}{\partial C_i} = \frac{\partial C_i}{\partial x}$

$= \frac{RT}{C_i} \frac{\partial C_i}{\partial x} = RT \frac{\partial \ln C_i}{\partial x}$

$$\Rightarrow \frac{\delta U_{gT}}{\delta x} = \frac{\delta u_i^{(0)}}{\delta x} - S_T \frac{dT}{dx} + V_i \frac{dP}{dx} + \frac{RT}{RT} \frac{\delta \ln C_i}{\delta x}$$

Example of deriving a "law"    
 Let  $u_g(x) = u_i^{(0)} + RT \ln C(x)$    
 one component, only concentration depends on  $x$ .   
 $\delta = \frac{1}{u} = \frac{RT}{D}$

$$\Rightarrow \frac{du_g}{dx} = RT \frac{\delta \ln C}{\delta x} = \frac{RT}{C} \frac{dC}{dx}$$

$$J = -UC \frac{du_g}{dx} = -URT \frac{dC}{dx} = -D \frac{dC}{dx}$$

$$\Rightarrow -\frac{dJ}{dx} = D \frac{d^2 C}{dx^2}$$

$$\Rightarrow \text{since } -\frac{dJ}{dx} = \frac{dC}{dt}$$

$$\frac{dC}{dt} = D \frac{d^2 C}{dx^2} \quad \text{Fick's law.}$$