Lecture 30 review: Random motion of particles: "Brownian" motion

$F_{random} \rightarrow \Delta v$ random $\rightarrow \Delta x$ random

$\xrightarrow{C.L.T.} \langle \delta x^2 \rangle = 2Dt$ ← $\frac{m^2}{s}$, $D = \frac{\langle \Delta x^2 \rangle}{\Delta t}$

↑ 1-Dimensional

ex: dermal patch

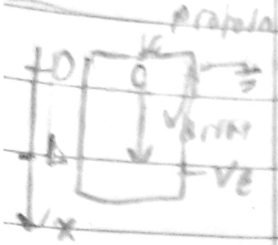
skin ≈ 2 mm

$t \approx 5$ hours

$D_{drug} \approx 100 \mu m^2/s$

$\langle 2000^2 \mu m^2 \rangle = 2 \cdot 100 \mu m^2/s \cdot t$

Lecture 31: Particle drift under external force $F_{applied}$



$V_\mathcal{E}(x) = x \cdot \mathcal{E}$ ← electric field, $\frac{V}{m}$

$V_\mathcal{E}(x=L) = V_\mathcal{E} = L \cdot \mathcal{E}$

- $F = ma = F_{applied} + F_{friction} + F_{random}$
- $F_{friction}$: $\bullet \xrightarrow{v_{drift}}$ friction ≈ # of collisions ↑ c
- ↑ $F_{friction}$ by ↑ v_{drift} or ↑ c.
- $F_{friction} = -\gamma v$ γ = friction coefficient ↑ c
  $u = \frac{1}{\gamma}$ = mobility
- $F_{applied}$: $E = \ldots + \mu_0 n + qV_\mathcal{E}$ ← $(C \cdot \frac{J}{C})$
- $\mu = \left(\frac{\partial E}{\partial n}\right)_{SN} = \mu_0 + \frac{\partial q}{\partial n} V_\mathcal{E}$

$= \mu_0 + zV_\mathcal{E}$ $z = \frac{\partial q}{\partial n} \left(\frac{C}{mole}\right)$ ↑ molar charge

- $F_{applied} = \frac{-\partial E}{\partial x}$

$= \frac{\partial}{\partial x}(q \cdot x \cdot \mathcal{E}) = -q \cdot \mathcal{E}$

- $F_{applied}$ per mole $= \frac{-\partial \mu}{\partial x} = z \cdot \mathcal{E}$

$\rightarrow \frac{-\partial \mu}{\partial x} + -\gamma v + F_{random} = ma \approx 0$

↑ true if friction of gel is large enough.

- $\frac{\partial \mu}{\partial x} = -\gamma v + F_{random}$ } Langevin Equation
- Let's solve this in two "limiting" cases.
- Case 1: assume $F_{appl} = 0$, $\mu = \mu_0$ = constant

$\gamma v = F_{random} \rightarrow \frac{\Delta x}{\Delta t} = \frac{F_{random}}{\gamma} \rightarrow \frac{\Delta x^2}{\Delta t} = \frac{F_{random} \Delta x}{\gamma}$

$\rightarrow \frac{\langle \Delta x^2 \rangle}{\Delta t} = \frac{\langle F_{random} \cdot \Delta x \rangle}{\gamma} \rightarrow D \approx \frac{\langle E_{random} \rangle}{\gamma}$

$\rightarrow D = \frac{k_B T}{\gamma}$ ← T is average energy per degree of freedom.

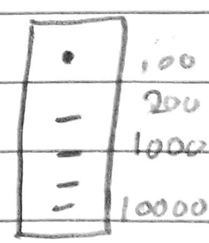
Brownian motion in 3-D: $\langle \delta x^2 \rangle = 6 \frac{k_B T}{\gamma} t$

- Case 2: Set $F_{random} = 0$, keep $\frac{\partial \mu}{\partial x}$ and $-\gamma v$

$\frac{\partial \mu}{\partial x} = -\gamma v = -z \cdot \mathcal{E} \rightarrow v_{drift} = \frac{z}{\gamma} \mathcal{E}$

$\gamma = 6\pi \cdot r \cdot \eta$

↑ radius of protein ↑ viscosity of solvent



does not depend on mass directly, as $m \propto r^3$, so bands get closer together.