L31: review

Molecule random motion

a) Central Limit theorem

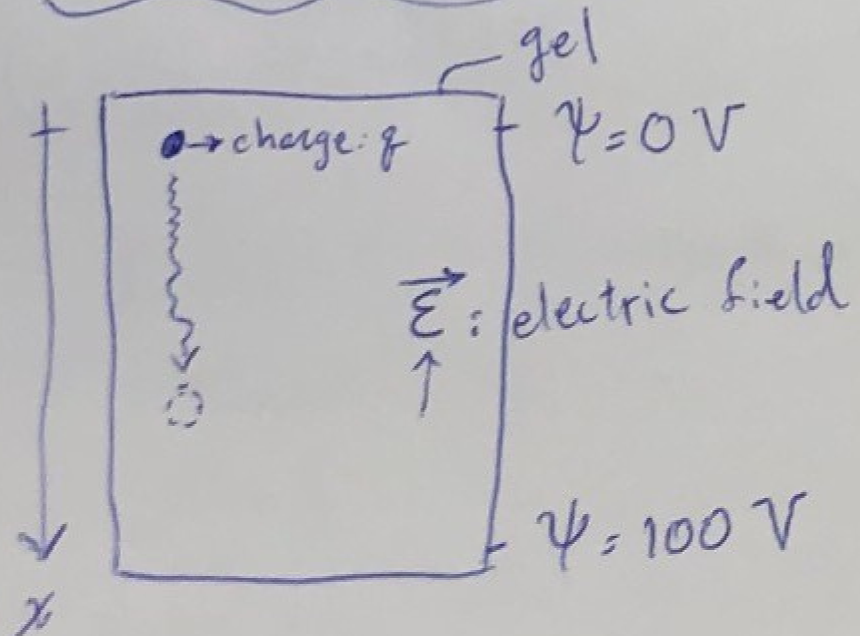
b) $F_{random} \Rightarrow v$ is random $\Rightarrow \Delta x$ is random

one-dimensional diffusion: $\underbrace{\delta x}_{\text{cumulative displacement}} = \sum_j \Delta x \Rightarrow \langle \delta x(t)^2 \rangle = 2\underbrace{D}_{\frac{\langle \Delta x^2 \rangle}{\Delta t}} t$

2-dimensional : $\langle \delta x(t)^2 \rangle = 4Dt$

3-dimensional : $\langle \delta x(t)^2 \rangle = 6Dt$

Directed Motion



$\Psi = -x \cdot \mathcal{E}$

$E = TS + \cdots + q\Psi$

$\mu = \mu^{\circ} + \cdots + q\Psi$

for the protein/DNA particle moving on gel: $F = F_{random} + F_{friction} + F_{applied} = ma$

$\boxed{\text{Today}}$ $F \simeq 0$ (high friction)

① $F_{applied} = -\frac{\partial E}{\partial x} = -\frac{\partial \mu}{\partial x}$

② $F_{friction} \sim -v \Rightarrow F_{friction} = -\gamma v$

proportional to velocity because friction is caused by the number of collisions of particle with the gel matrix in the unit of time; faster movement causes more collisions.

① and ②: $F = F_{random} - \gamma v - \frac{\partial \mu}{\partial x} = 0$

$\Rightarrow \frac{\partial \mu}{\partial x} = F_{random} - \gamma v$

Case 1: $\mu = \text{constant} \Rightarrow \frac{\partial \mu}{\partial x} = 0 \Rightarrow v = \frac{F_{random}}{\gamma}$

$\Rightarrow \frac{\partial x}{\partial t} = \frac{F_{random}}{\gamma} \Rightarrow \frac{\Delta x}{\Delta t} \approx \frac{F_{random}}{\Delta t}$

$\xrightarrow{\times \Delta x} \frac{\Delta x^2}{\Delta t} = \frac{\overbrace{F_{random} \cdot \Delta x}^{\Delta E_{random}}}{\gamma}$

* it can be shown that ΔE_{random} is proportional to E_{random}. This is a result of linear response theory:

$$\Delta E_{random} \sim E_{random}$$

$$\hookrightarrow \frac{\Delta x^2}{\Delta t} \sim \frac{E_{random}}{\gamma} \Rightarrow \frac{\langle \Delta x^2 \rangle}{\Delta t} \sim \frac{\langle E_{random} \rangle}{\gamma}$$

$$\Rightarrow D \sim \frac{k_B T}{\gamma}$$

it can be shown that proportionality constant is $\underset{-}{1}$

$$D = \frac{k_B T}{\gamma} \; ; \; \gamma = \frac{k_B T}{D} \equiv \frac{1}{u}$$

γ → friction coefficient

D → diffusion coefficient

u → mobility

Case1

$$\boxed{\Rightarrow \text{Diffusion law: } \langle \delta x(t)^2 \rangle = 2 \cdot \frac{k_B T}{\gamma} \cdot t}$$

Case 2: Neglect random Force

$$\text{if } \mu = \mu_0 - q \cdot x \cdot \mathcal{E} \Rightarrow \frac{\partial \mu}{\partial x} = -q \cdot \mathcal{E}$$

$$\Rightarrow \frac{\partial \mu}{\partial x} = -\gamma v = -q \mathcal{E} \Rightarrow v = const \equiv v_{drift}$$

$$v_{drift} = \frac{q \mathcal{E}}{\gamma}$$

larger particles with equal charge drift slower because they have a higher friction coefficient.

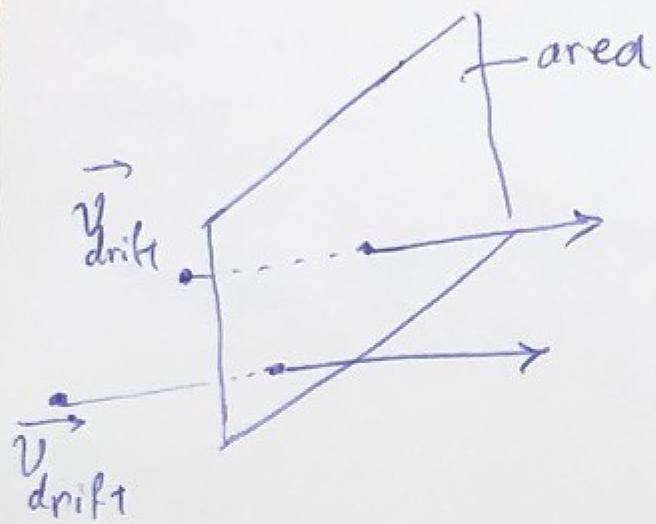
Often (roughly spherical objects):

$$\gamma = \frac{1}{u} = 6\pi \cdot R \cdot \eta$$

R → radius of molecule

η → viscosity of gel (unit: Pa-s)

Flux & continuity equation



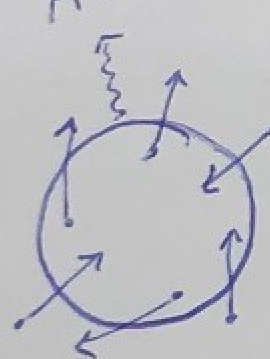
$$\overset{\text{flux}}{\vec{J}}\left(\frac{\text{moles}}{m^2\cdot s}\right) = \overset{m/s}{v_{drift}} \cdot \overset{\text{moles}/m^3}{C}$$

also: $\frac{\partial \mu}{\partial x} = -\gamma v_{drift}$

$$v_{drift} = -\frac{1}{\gamma}\frac{\partial \mu}{\partial x} = -u\frac{\partial \mu}{\partial x};$$

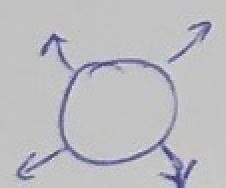
$$J = -uC\underbrace{\frac{\partial \mu}{\partial x}}_{\text{driving force, e.g. } -q\mathcal{E}}$$

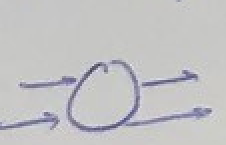
A: surface area



$$\iiint dV\cdot\frac{\partial C}{\partial t} = \iint d\vec{A}\cdot\vec{J}$$

Gauss's theorem: $\iint d\vec{A}\cdot\vec{J} = \iiint dV\underbrace{\vec{\nabla}\cdot\vec{J}}_{\text{divergence of flux}}$

 $\vec{\nabla}\cdot\vec{J} \neq 0$

 $\vec{\nabla}\cdot\vec{J} = 0$

$$\Rightarrow \frac{\partial C}{\partial t} = \frac{\partial J}{\partial x} \quad \text{(continuity of flux)}$$