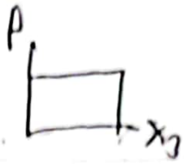
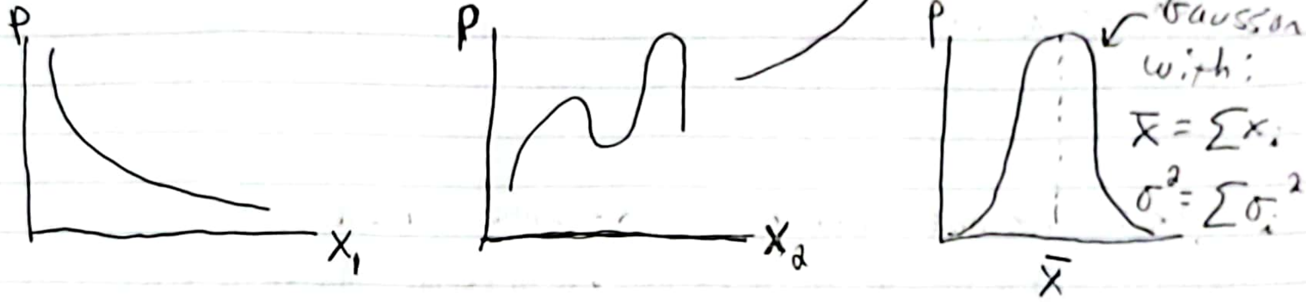
Lecture 30



Review from long ago: CLT (central limit theorem)

$X_1 + X_2 + X_3 + \ldots = X$



Today: Time-dependent stat mech: kinetics and transport

① Brownian motion

Short time: F=ma predicts where particles go.

Long time: "Butterfly effect" force behaves randomly

$V = \frac{\Delta x}{\Delta t}$; $F = ma = m\frac{\Delta V}{\Delta t}$ $\Rightarrow \left[\Delta x = V \cdot \Delta t ; \quad \Delta V = \frac{F}{m}\Delta t\right]$

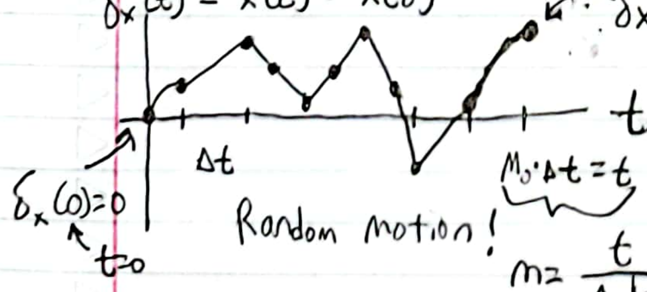
$x(t+\Delta t) = x(t) + V \cdot \Delta t$; $V(t+\Delta t) = V(t) + \frac{F(t)}{m} \cdot \Delta t$

We can use this to propagate the position of a particle in time.

What if $F(t)$ is a random force?

$\delta_x(t) = x(t) - x(0)$

$\delta_x(t) = \delta_x(m \cdot \Delta t) = \sum_{j=1}^{m} \Delta x$



$m = \frac{t}{\Delta t}$

$\Rightarrow \delta_x(t)$ is a gaussian-distributed random variable with standard deviation given by $\langle \delta x^2 \rangle = \sum_{j=1}^{m} \Delta x_j^2$

$\Rightarrow \langle \delta x^2 \rangle = m \cdot \langle \Delta x^2 \rangle = \frac{t}{\Delta t} \cdot \langle \Delta x^2 \rangle = \underbrace{\frac{\langle \Delta x^2 \rangle}{\Delta t}}_{\text{constant}} \cdot t$; $\frac{\langle \Delta x^2 \rangle}{\Delta t} = 2D =$ constant finally:

$\langle \delta x^2 \rangle = 2 \cdot D \cdot t$ law of diffusion in 1-D

$\delta x_{rms} = \sqrt{\langle \delta x^2 \rangle} = \sqrt{2D \cdot t}$

In 3-D: $\langle \delta r^2 \rangle = 6 \cdot D \cdot t$

ex = dermal patch, $\Delta x = 2\,mm$ of skin

$D = 100\ \mu m^2/s$ (typical for organic molecule)

$t = \frac{\langle \Delta x^2 \rangle}{2D} = \frac{(2000\ \mu m)}{2 \cdot 100\ \mu m^2/s} = 20{,}000\ s \approx 5\ hours$