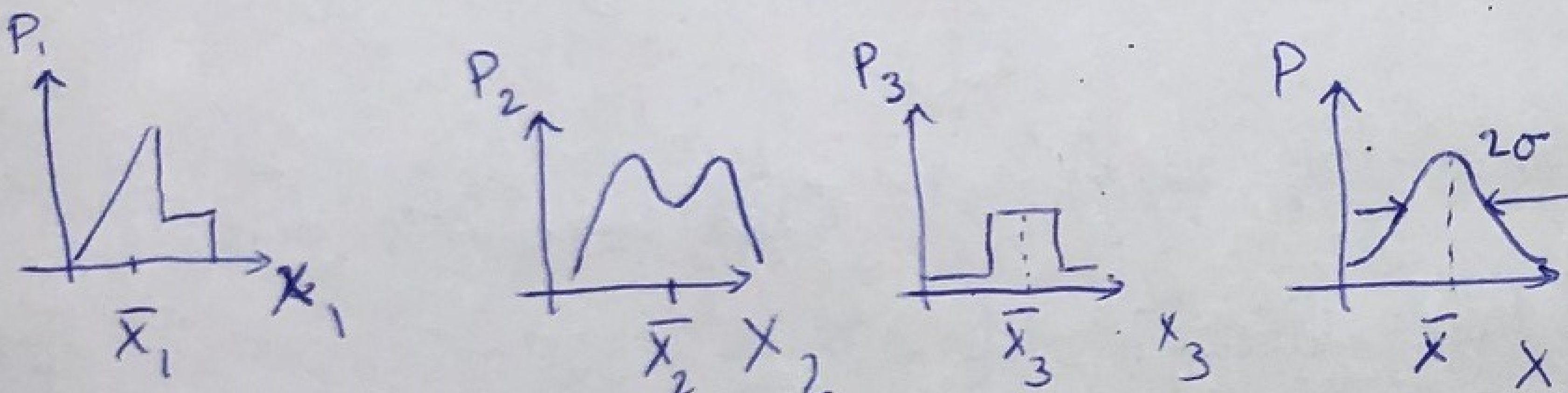
L30: review

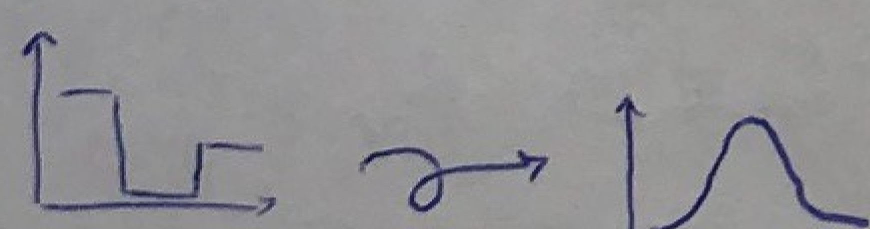
Central limit theorem (C.L.T)

$X_1 + X_2 + X_3 + \cdots = X$ ← random variable




$X$ is gaussian distributed with
$$\begin{cases} \bar{X} = \sum_j \bar{x}_j & \text{(average)} \\ \sigma^2 = \sum_j \sigma_j^2 & \text{(standard deviation)} \end{cases}$$

* Prof. Gruebele showed a demo of non-normal distributions becoming normal if multiple numbers are picked from the non-normal distribution and added together: (and this repeated thousands of time)



non-normal distribution → normal distribution

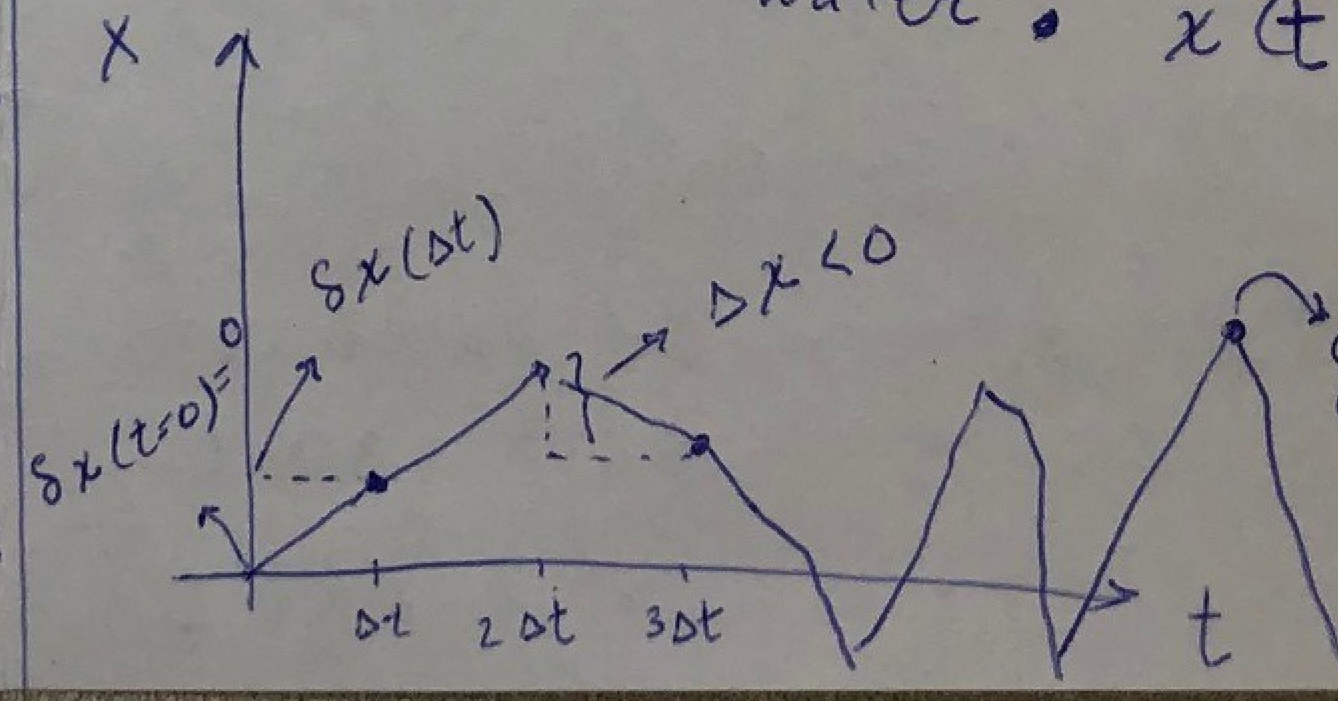
Today: How molecules move to get to equilibrium

Brownian motion

At short times, any motion is predictable by Newtonian mechanics:

$$X(\Delta t) = X(0) + V(0)\cdot\Delta t$$
$$F = ma = m\frac{\Delta V}{\Delta t} \Rightarrow \Delta V = \frac{F}{m}\Delta t$$
$$\Rightarrow V(\Delta t) = \frac{F(0)}{m}\Delta t + V(0)$$

if F (from thermal motion) is random, e.g in the Brownian motion of a dust particle in water: $x(t)$ = random walk




$$\delta x(t) = \sum_{i=1}^{m} \Delta x_j$$

random variables

No matter what the probabilitiy distributions of the $\Delta x_j$ look like, for sure $\delta x(t)$ will be gaussian distributed after enough steps:

$$\delta x = \sum_{j=1}^{m} \Delta x_j \quad , \quad m = \frac{t}{\Delta t}$$

C.L.T

$$\langle \delta x^2 \rangle = m \langle \Delta x^2 \rangle$$

$$\langle \delta x^2 \rangle = \underbrace{\frac{\langle \Delta x^2 \rangle}{\Delta t}}_{\text{Constant}} \cdot t$$

$$\langle \delta x^2 \rangle = 2D \cdot t$$

$\Rightarrow$ root mean squared displacement $\sqrt{\langle \delta x^2 \rangle} \sim t^{1/2}$

D: diffusion coefficient

1-dimentional diffusion: $\langle \delta r^2 \rangle = 2D \cdot t$
2- " " : $\langle \delta r^2 \rangle = 4D \cdot t$
3- " " : $\langle \delta r^2 \rangle = 6D \cdot t$

Exercise: drug from patch has to diffuse through skin (e.g. nicotine patch)

$\sqrt{\langle \delta x^2 \rangle} \simeq 2\text{ mm}$ ; $D \sim 100\ \mu m^2/s$

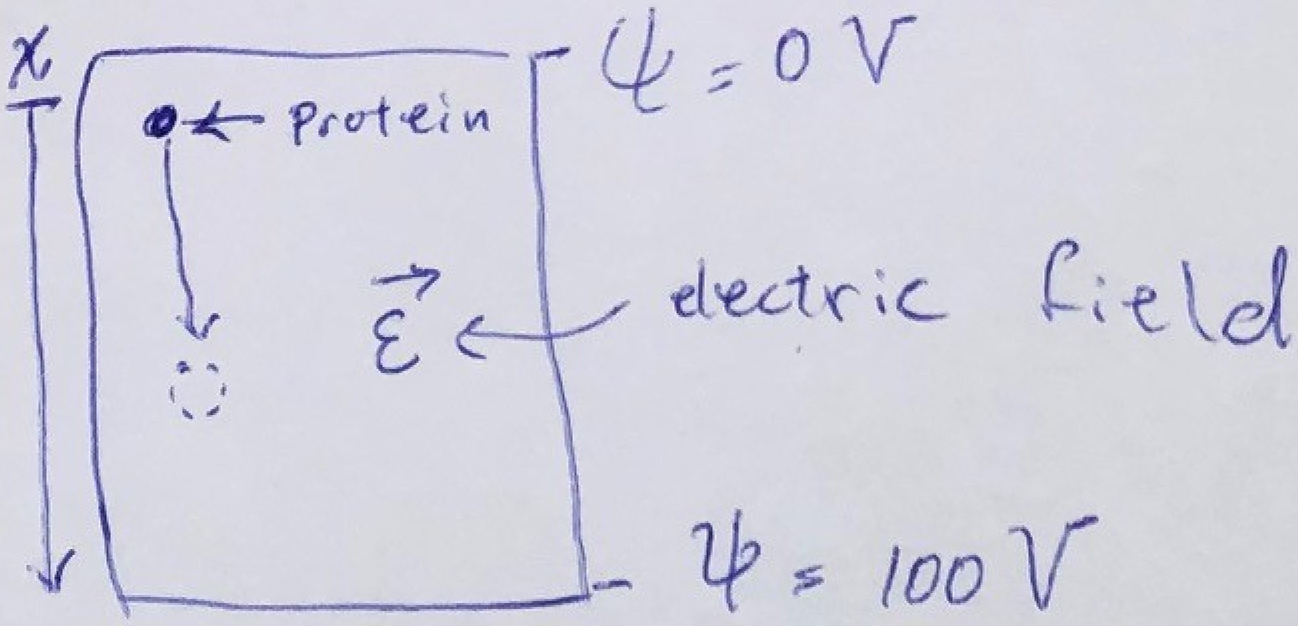
how long does it ~~tag~~ take for drug to diffuse through skin?

$$\langle 2000\ \mu m \rangle^2 = \underset{\substack{\downarrow \\ \text{3-D diffusion}}}{6} \cdot 100\ \mu m^2 s^{-1} \cdot t$$

$\Rightarrow t = 6666.6\ldots$ seconds $\approx 1.85$ hours

* diffusion is also observed in gel electrophoresis. As protein or DNA sample is drifting through the gel (induced by Voltage difference), the band starts to diffuse slowly. Hence, the voltage needs to be high enough that the sample does not diffuse away too much by the end of the experiment.

# Drift Velocity:




voltage

from stat mech: $E = ST + \cdots + \underbrace{q\psi}_{-q \cdot x \cdot \varepsilon}$

for one species (protein): $G = \mu(x) = ST + \cdots - qx\varepsilon$

$F = ma$

$\underbrace{F_{\text{applied}}}_{\text{electric field}} + F_{\text{friction}} + F_{\text{random}} = ma$