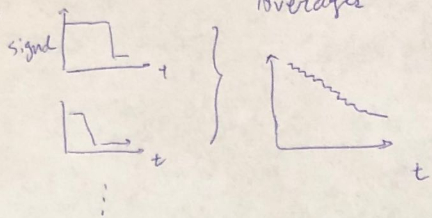
L3: Review

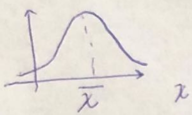
1. Averages



2. Diff eq

$$\frac{\partial N}{\partial t} \sim N$$

3. Central limit theorem



$x = \sum x_i$ and $\bar{x} = \sum \bar{x}_i$

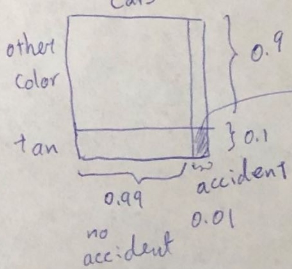
4. Bayes theorem

$$P(A|B) = P(B|A)\frac{P(A)}{P(B)}$$

today: 5 and 6 from "O" Notes.

5 Logs: multiplying → adding

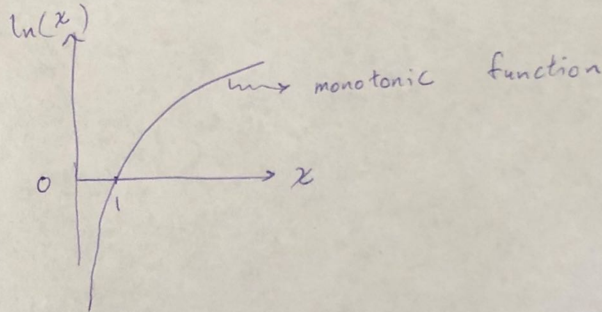
example:



$P(A+B) = 0.1 * 0.01 = 0.001 = 10^{-3}$

To compress the dynamic range of probabilities we can instead use the exponent (-3):

$\underline{\log}(10^{-3}) = -3$



$z = x \cdot y \Rightarrow \ln(z) = \ln(x) + \ln(y)$

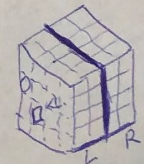
multiplication

addition

⇓

$\ln(z)$ is an "extensive" variable

ex:



3 molecules

$W_{1/2}$: possibilities of all three molecules in the left (or right) half compartment

○: 32 ways of placing

△: 31

□: 30

$W_{Y_2} = 32 \cdot 31 \cdot 30 = 29\,760$

$W_{tot} = 64 \cdot 63 \cdot 62 = 249\,984$

$P = \frac{W_{Y_2}}{W_{tot}} \sim 12\%$

$\frac{W_{Y_2}}{W_{tot}} \xrightarrow{\ln} S = \ln W$

$\downarrow$

becomes $S_{Y_2} - S_{tot} \longrightarrow S_{tot} = \ln W_{tot} = 12.4$

$\swarrow$

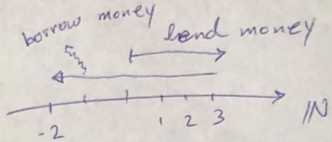
$S_{Y_2} = \ln W_{Y_2} = 10.3$

S is an extensive (additive) variable

$S \equiv$ entropy

6. Complex numbers

MWG history of numbers:



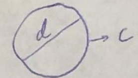
$3 - 5 = -2$

negative values denote money owed

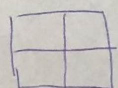
rational numbers

$\frac{5}{10} = \frac{1}{2}$

irrational numbers:



$d \sim c \qquad \frac{c}{d} = \pi$

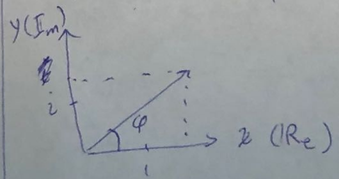


$\left.\begin{array}{l} A = 4 \\ L = 2 \end{array}\right\} \; L = \sqrt{A} = \sqrt{4}$

what about $\sqrt{-4}$?

$\sqrt{-4} = \sqrt{4}\,\sqrt{-1} \rightsquigarrow i$: imaginary number

graphical representation:



imaginary component is shown on the vertical axis and the real component on the horizontal ex: $z_1 = x_1 + iy_1$, $z_2 = x_2 + iy_2$ $\Big\}$ $z = (x_1 + x_2) + i(y_1 + y_2)$

$= x + iy$

ex: $z_1 \cdot z_2 = (x_1 + iy_1)(x_2 + iy_2) =$

$x_1 x_2 - y_1 y_2 + i(x_1 y_2 + x_2 y_1)$

Taylor series: polynomial calculation ab functions

examples:

$$e^x = 1 + x + \frac{1}{2!} x^2 + \frac{1}{3!} x^3 + \cdots$$

$$\cos \varphi = 1 - \frac{1}{2!} \varphi^2 + \frac{1}{4!} \varphi^4 - \cdots$$

$$\sin \varphi = \varphi - \frac{1}{3!} \varphi^3 + \cdots$$

Prove that: $\boxed{e^{i\varphi} = \cos \varphi + i \sin \varphi}$

$$e^{i\varphi} = 1 + i\varphi + \frac{1}{2!}(i\varphi)^2 + \frac{1}{3!}(i\varphi)^3 + \cdots$$

$$\cos\varphi + i \sin \varphi = 1 + i\varphi - \frac{1}{2!}\varphi^2 - \frac{1}{3!} i\varphi^3$$