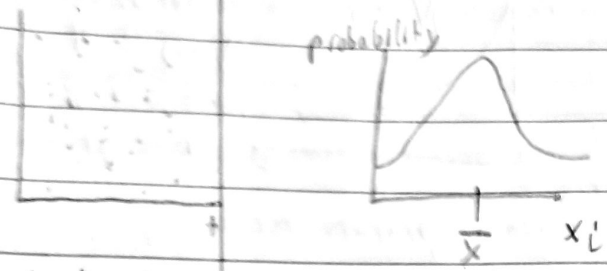
Lecture 2: Randomness

$X(t) = x_1(t) + x_2(t) + x_3(t) + \dots x_n(t)$, $n \to \infty$

$x_i(t)$ are random, independent variables.

But $x(t)$ will approach being gaussian



Central Limit Theorem:

The sum of independent random variables will approach a Gaussian distribution as

$\bar{x} = \bar{x}_1 + \bar{x}_2 + \bar{x}_3 + \dots$ the # of variables ↑.

$s^2 = s_1^2 + s_2^2 + s_3^2$

Ex: Dice

Die 1 = x_1

flat distribution

1 2 3 4 5 6

Die 1 + Die 2: $x = x_1 + x_2$

triangular distribution

2 3 4 5 6 7 8 9 10 11 12 (possible rolls)

$\frac{1}{36}, \frac{1}{18}, \frac{1}{12}, \frac{1}{9}, \frac{5}{36}, \frac{1}{6}, \frac{5}{36}, \frac{1}{9}, \frac{1}{12}, \frac{1}{18}, \frac{1}{36}$ (probability)

"inference" →

Bayesian Inference

The statistics of "what if"

P(A): probability that A happens

P(B): probability that B happens

P(A|B): probability that A happens if B is true.

P(B|A): probability that B happens if A is true.

$P(A \cap B)$: probability that A & B both happen.

$P(A|B) = \frac{P(A \cap B)}{P(B)}$ $P(B|A) = \frac{P(B \cap A)}{P(A)}$

$P(A \cap B) = P(B \cap A)$ (A∩B) (B∩A)

$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$ (conditional probability)

Ex. Horse racing

Horse wins 1 out of 4 races. $P(B) = \frac{1}{4}$

The time he won, it rained. $P(A) = \frac{1}{4}$

For a fifth race, will he win if it rains?

$P(B|A) = \frac{P(A|B)\,P(B)}{P(A)} = \frac{1 \cdot \frac{1}{4}}{\frac{1}{4}} = 1$ Yes

Say Hunt loses in race 5, with rain.

$P(B) = \frac{1}{5}$ $P(A) = \frac{2}{5}$

$P(A|B) = 1$ (The only time he won it was in rain)

$P(B|A) = \frac{1 \cdot \frac{1}{5}}{\frac{2}{5}} = 0.5$

Note that these are always just predictions. Especially with little data, they can fail. When it does, that updates the data, making it less likely to fail.