Lecture 2 8/23/23

Last time:

How physicists think: ex (2) "rate ~ number of molecules"

a) Postulate

⇓ math

b) Model (e.g. DEQ) ⇒ $\frac{\partial N}{\partial t} \sim N$

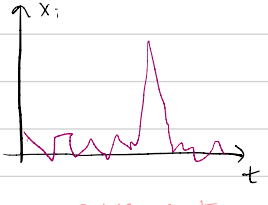
⇓ solve for desired eq.

c) Equation for property or interest ⇒ $N(t) = N_0 e^{-kt}$ w/ k positive, constant

Today: CLT & Bayesian Inference

(3) Randomness

ex:

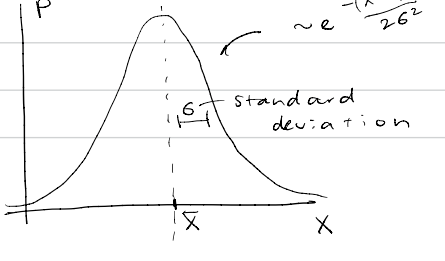


measurement of x_i over time

probability distribution (how likely to get x_i)

Central Limit Thm:

$$X(t) = X_1(t) + X_2(t) + \dots + X_n(t)$$



As $n \to \infty$, probability distribution approaches a guassian.

$$\bar{X} = \bar{X}_1 + \bar{X}_2 + \dots$$

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 + \dots$$

④ Bayesian Inference: (Thomas Bayes)

- The statistics of "what if" (conditional probability)

Ex: Coin toss

T H T T H H T H T H T H T H H T T H ...

T T H H T T H H T T H H T T H H ...

(will H happen if 2 Ts just happened?

Bayes' Formula:

$$P(A|B) = P(B|A) \cdot \frac{P(A)}{P(B)}$$

$P(A|B)$: prob. A happens if B is true

$P(B|A)$: prob. that B happens if A is true

$\frac{P(A)}{P(B)}$: prob. that A happens / prob that B happens

Ex: Using conditional probabilities

Lauda won 3 out of 4 times against Hunt. Who would you bet on and at what odds?

Let $P(B) = P(\text{Hunt wins}) = 1/4$

So odds are 3:1 in favor of Lauda

What if Hunt drives better in rain and the 1/4 times he won, it rained, and the forecast for race #5 is heavy rain?

Who to favor?

Let A = "it rains", B = "Hunt wins"

$$P(B|A) = P(A|B) \cdot \frac{P(B)}{P(A)}$$

$$= 1 \cdot \frac{0.25}{0.25} = 1 \Rightarrow \text{bet on Hunt.}$$

- You can always update conditional probability.

Let's say Hunt lost race #5.

Then:

$$P(B|A) = P(A|B)\frac{P(B)}{P(A)}$$

$$= 1 \cdot \frac{0.2}{0.4} = 0.5$$