

Bonding in H_2 : molecular orbitals (MO)

Next 2 lectures: the most common basis sets used for complete accurate electronic structure calculations

Today: MOs

Next: Valence bonds (VBs)

$$\hat{H} = h_1(\vec{r}_1) + h_2(\vec{r}_2) + \frac{e^2}{4\pi\epsilon_0 r_{12}} + \frac{e^2}{4\pi\epsilon_0 R_{12}} - \text{sec N26}$$

← H-atom ← H-atom ← e-e repulsion
 (little r is distance between e's)
 ↑ nuclear repulsion

Smallest basis: $|s_A\rangle | \pm \frac{1}{2} \rangle$ and $|s_B\rangle | \pm \frac{1}{2} \rangle$ [4 functions]
 ↑ approximate
 ← spin α or β

Using what we learned about H_2^+ , combine $|s_A\rangle$ and $|s_B\rangle$ into a new basis

$$\begin{aligned} |\sigma^+ \rangle &= \frac{1}{\sqrt{2}} \{ |s_A\rangle - |s_B\rangle \} | \pm \frac{1}{2} \rangle \\ |\sigma \rangle &= \frac{1}{\sqrt{2}} \{ |s_A\rangle + |s_B\rangle \} | \pm \frac{1}{2} \rangle \end{aligned}$$

Minimally we need 2 basis functions, if neglected high energy orbitals

The actual basis functions must be antisymmetric:

$$\begin{aligned} \psi(\vec{r}_1, \vec{r}_2, S=0) &\cong |1\rangle = \frac{1}{\sqrt{2}} \begin{vmatrix} |s_A\rangle \alpha_1 & |s_A\rangle \beta_1 \\ |s_B\rangle \alpha_2 & |s_B\rangle \beta_2 \end{vmatrix} \\ &= \frac{1}{\sqrt{2}} \{ |s_A\rangle |s_B\rangle \alpha_1 \beta_2 - |s_A\rangle |s_B\rangle \alpha_2 \beta_1 \} \\ &= \frac{1}{\sqrt{2}} |s_A\rangle |s_B\rangle \underbrace{\{ \alpha_1 \beta_2 - \alpha_2 \beta_1 \}}_{\text{antisymmetric part of the spin function}} \end{aligned}$$

$S=0$ has only one orientation: $S_z = M_s = 0$
 ↑ state
 we derive from s orbitals, no nodes

both e⁻ in bonding orbital
 ⇒ lowest E
 ⇒ Pauli exclusion: must have opposite spin

An example of a higher energy basis function:

$$\Psi_L(r_1, r_2, s=1) \hat{=} |2\rangle = \frac{1}{\sqrt{2}} \begin{vmatrix} |1\sigma\rangle_1 \alpha_1 & |1\sigma^*\rangle_1 \alpha_1 \\ |1\sigma\rangle_2 \alpha_2 & |1\sigma^*\rangle_2 \alpha_2 \end{vmatrix}$$

$S=1$ has $M_S = \pm 1, 0, -1$
 \sum state
 3 orbitals
 e- must be in different spatial orientation orbitals \neq same spin

$$\hat{H} = \begin{pmatrix} H_{11} & 0 & \dots \\ 0 & H_{22} & \\ \vdots & & H_{nn} \end{pmatrix} \rightarrow H_{12} = 0 \text{ (too large)}$$

very high energy, can leave out

