Lecture 21 review

$W(E,V,n\ldots) \xrightarrow{\ln} S(E,V,n\ldots) \xrightarrow{\text{invert}} E(S,V,n\ldots) \rightarrow dE = \underbrace{\left(\frac{\partial E}{\partial S}\right)}_{T>0} dS + \underbrace{\left(\frac{\partial E}{\partial V}\right)}_{-P} dV + \underbrace{\left(\frac{\partial E}{\partial n}\right)}_{\mu} dn + \ldots$

↑ extensive variables

isolated sys

heat flow; $dE_1 = -dE_2$	$dE=0$	$T_1 = T_2$ when energy can equilibrate between the two sides.

Isolated: $dV=0$

volume; $dV_1 = -dV_2$	$P_1 = P_2$ when volume can equilibrate between the two sides

Lecture 22: E, F, G, H

- E, F, G, H...: a series of functions that contain identical info but depend on different variables.
- Simple formula: extensive $\rightarrow E(\lambda S, \lambda V, \ldots)$
  (if $2S, 2V$ ..., $2E$) $\rightarrow \lambda E(S,V,\ldots)$
  - taking derivative with respect to λ.

$$\frac{\partial E}{\partial \lambda S}\frac{\partial \lambda S}{\partial \lambda} + \frac{\partial E}{\partial \lambda V}\frac{\partial \lambda V}{\partial \lambda} + \ldots = \frac{\partial}{\partial \lambda}\lambda E(S,V,\ldots)$$

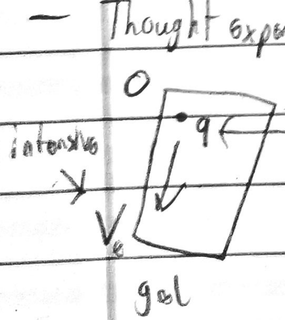
$$\left(\frac{\partial E}{\partial \lambda S}\right)S + \left(\frac{\partial E}{\partial \lambda V}\right)V + \ldots = E(S,V,\ldots)$$

- true for any λ, including $\lambda = 1$

$$\frac{\partial E}{\partial S}S + \frac{\partial E}{\partial V}V + \left(\frac{\partial E}{\partial n}\right)n = E(S,V,n)$$

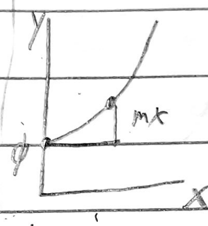
$$TS + -PV + \mu n + \ldots = E(S,V,n)$$

- Thought experiment: other variables



$$E = TS - PV + \mu n + V_0 \cdot q + \ldots$$

- Why F, G, H...: $E(S,V,n)$ has the same info, but S, V... may not be convenient.
- Why not just solve $E(S) \rightarrow T = \frac{\partial E}{\partial S} = T(S)$ ⟶ invert
  Doesn't work. ⟶ $E(T) = E(S(T)) \leftarrow S(T)$



Function	Variable	slope	intercept
y	x	$\frac{\partial y}{\partial x} = m$	$\phi = y - mx$
E	S	$\frac{\partial E}{\partial S} = T$	$F = E - ST$

$\phi = y - mx$

- ~~E(T) or y(m) loses info about the function~~
- ~~$z(m) = y(m)$ even though $z(x) \neq y(x)$~~
- $y(x) = x^2$ and $z(x) = (x-5)^2$

$\frac{\partial y}{\partial x} = m = 2x$ $\qquad$ $\frac{\partial z}{\partial x} = m = 2(x-5)^2$

$x = m/2$ $\qquad$ $(x-5) = m/2$

$y(x(m)) = \frac{m^2}{4}$ $\qquad$ $z(x(m)) = \frac{m^2}{4}$

- What we need instead of E(T), which loses info, is a function F(T) that is equivalent but preserves info. $y(x) \rightleftarrows \phi(m)$

$y(x) \rightarrow y(m)$ can't go back

$$E(S,V,n) \xrightarrow[\text{to } T]{\text{from } S} F(T,V,n) = E - TS \xrightarrow{\text{from } V \text{ to } P} G(T,P,n) = E - TS + PV = H - TS$$

$\xrightarrow{\text{from } V \text{ to } P} H(S,P,n) = E + PV$ ⟶ from S to T