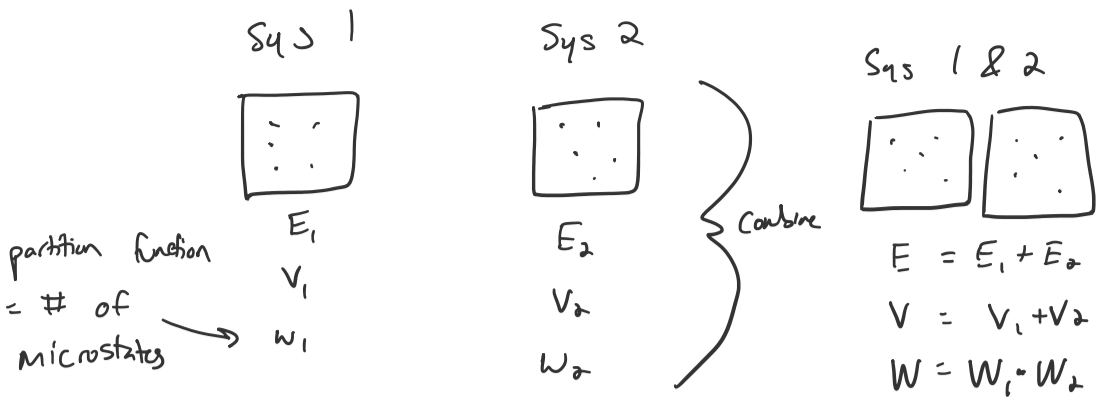


Lecture 20

Monday, October 9, 2023 9:56 AM

Last time = W & S , multiplicative vs. additive



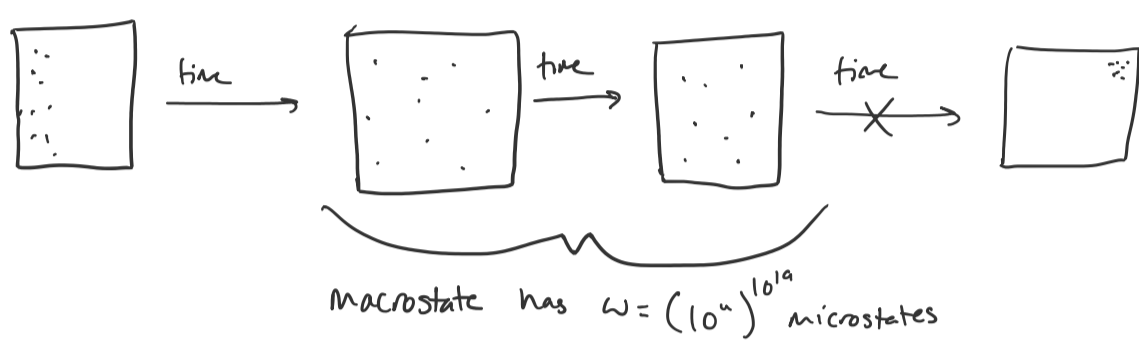
Define: $S = \ln W$ or $S = k_B \ln W$, where k_B is any positive number

$\Rightarrow S = S_1 + S_2$ extensive ($\ln(ab) = \ln a + \ln b$)

$\Rightarrow W$ can be retrieved from S (\ln is a monotonic function)

$P = \frac{W_1}{W_2} \Rightarrow \ln P = \ln \frac{W_1}{W_2} = S_1 - S_2 = \Delta S$
 < 0 if $w_1 < w_2$

Today: more on S ; 2nd & 3rd Law



$W(T > 0) > W(T = 0)$ (equality holds if the macrosystem already occupies typical microstates)

$\Rightarrow S(T > 0) > S(T = 0)$ or $\Delta S = S(T > 0) - S(T = 0) \geq 0$ 2nd law

Why multiply by k_B ? k_B = Boltzmann's Constant

In 19th century, it was not realized that "temperature" = "energy per degree of freedom"

"Temperature" \sim "Energy per molecule"

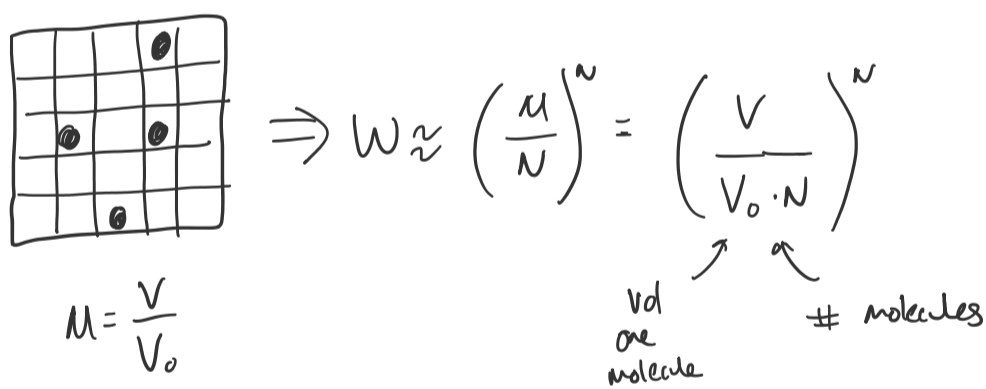
Because of this, $\frac{E}{T}$ has units $\frac{\text{Joules}}{\text{kelvin}}$

$0 \text{ K} \rightarrow 0 \text{ J}$

$1 \text{ K} \rightarrow 1.38 \times 10^{-23} \text{ Joule} \Rightarrow k_B = 1.38 \times 10^{-23} \text{ J/K}$

For a mol, $S = A \cdot k_B \cdot \ln W = 6.02 \times 10^{23} \times 1.38 \times 10^{-23} \ln W$
 $= R \ln W$ $R = 8.314 \text{ J/mol}\cdot\text{K}$

ex. HW S2.3 Entropy of a gas in a box



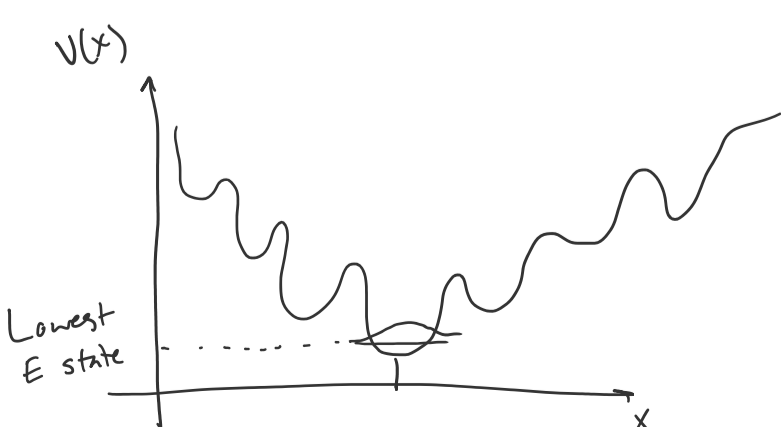
$S \approx k_B N \ln \left(\frac{V}{V_0 \cdot N}\right)$
 $\approx N \cdot k_B \left(\ln \frac{V}{N} - \ln V_0\right)$
 $= N k_B \ln \frac{V}{N} - N k_B \ln V_0$
 $= A n k_B \ln \frac{V}{N} - A n k_B \ln V_0$
 $= n R \ln \frac{V}{N} - n R \ln V_0$

S of gas in box is extensive

In PQ, $p_j = \frac{1}{W}$

$S = k_B \ln W = -k_B \omega \frac{1}{\omega} \ln \left(\frac{1}{\omega}\right)$
 $= -k_B \omega p_j \ln p_j$
 $= -k_B \sum_{j=1}^{\omega} p_j \ln p_j$

More microstates \Leftrightarrow more likely \Leftrightarrow higher entropy



$\lim_{E \rightarrow \min} W = 1$ $\lim_{E \rightarrow \min} S = 0$ 3rd law