Last Time:

P1: Laws of motion apply to an isolated as $N \to \infty \Rightarrow E$ is conserved ("1st law")

P2: In an isolated system, all microstates (The "micro-canonical ensemble") are equally probable.

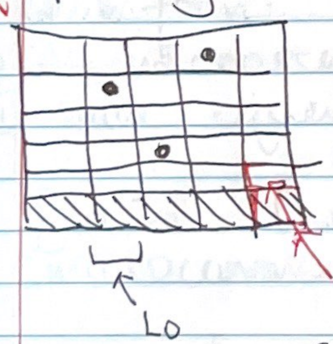
$$P_j = \frac{1}{W}$$

where the partition function $W = \#$ of accessible microstates.

P2 Strong: Time accuracy = ensemble average.

Today: W and S.

Calculation - why P2 - strong must strictly speaking fail.



$V_0 = L_0^3$

$N = 3$

$M = \frac{V}{V_0} = 25$

$M >> N$

$$W = \frac{25 \times 24 \times 23}{3!} = \frac{M!}{(M-N)!\,N!}$$

Grand approx: $N! \simeq N^N e^{-N}$ if $N >> 1$

ex: $M >> N >> 1$

$$\Rightarrow \frac{M!}{(M-N)!\,N!} = \frac{M^M e^{-M}}{(M-N)^{M-N} e^{-M+N} \cdot N^N e^{-N}}$$

$$= \frac{M^M}{\underbrace{(M-N)}_{M}^{M-N} \, N^N}$$

$$\simeq \frac{M^M}{M^{M-N} \, N^N} = \left(\frac{M}{N}\right)^N$$

Let's plug in some realistic numbers:

$V = 1\,cm^3$ of gas at 1 atm and room T. has $\simeq 10^{19}$ molecules. Assuming each molecule has a volume $V_0 \simeq 10\,Å^3$, moving at $v \simeq 300\,m/s$

$$M = \frac{V}{V_0} = \frac{1\,cm^3}{10\,Å^3} = 10^{23} \qquad N = 10^{19}$$

$$\Rightarrow \frac{M}{N} = 10^4 \qquad W = (10^4)^{10^{19}}$$ = number of microstates for $1\,cm^3$ of gas.

How many of these microstate have been visited since the begining of the universe?

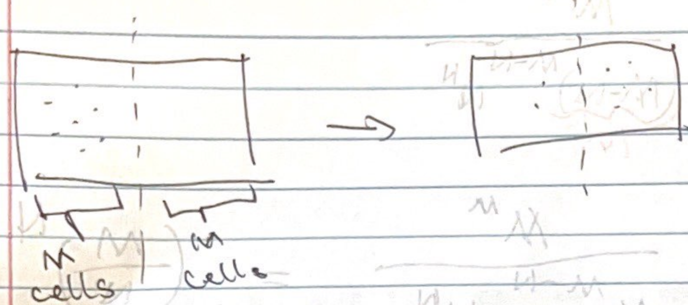
$L_0 = V_0^{1/3}$ and $v_{gas} = 300\,m/s$

$$\Rightarrow \Delta t = \frac{L_0}{v_{gas}} \simeq 10^{-12}\,s = 1\,ps$$

Universe lifetime $\simeq 10^{18}\,s$!

$$\Rightarrow W_{actual} \simeq \frac{10^{18}\,s}{10^{-12}\,s} = 10^{30}$$

Another calc. with $W =$



$W_{left} \approx \left(\frac{M}{N}\right)^N$

$W_{whole} \approx \left(\frac{2M}{N}\right)^N$

$P_{left} = \frac{W_{left}}{W_{whole}}$ by P2 $\Rightarrow P_{left} = \left(\frac{1}{2}\right)^N$

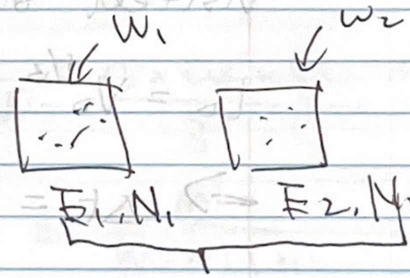
For $N = 6 \Rightarrow \left(\frac{1}{2}\right)^6 = 0.015 = 1.5\%$

For $N = 10^{19} \Rightarrow \left(\frac{1}{2}\right)^{10^{19}}$

$W(t>0) \geq W(t=0) \quad N \to \infty$

W and S.

For 2 isolated systems



$E_{whole} = E_1 + E_2$ (P1)

$N_{whole} = N_1 + N_2$

$W = W_1 \cdot W_2 \rightarrow$ multiplicative and very large

Solution: Define $S = k_B \ln W$

$\Rightarrow S = S_1 + S_2$