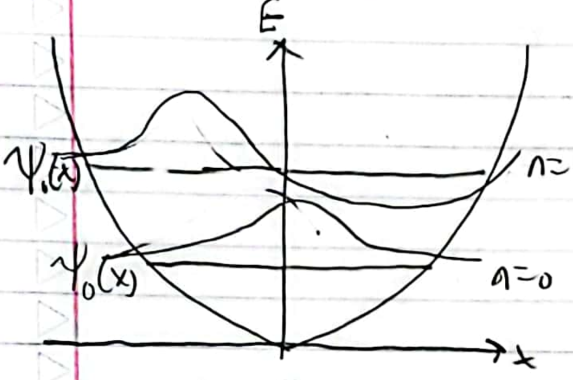
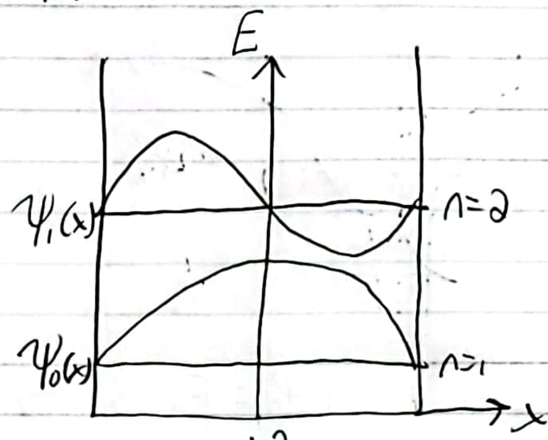
Lecture 10

- Last time: Quantum energy Levels



$$E_n = \frac{h}{2\pi}\sqrt{\frac{k}{m}}\left(n+\frac{1}{2}\right)$$

Spring vibrating bond

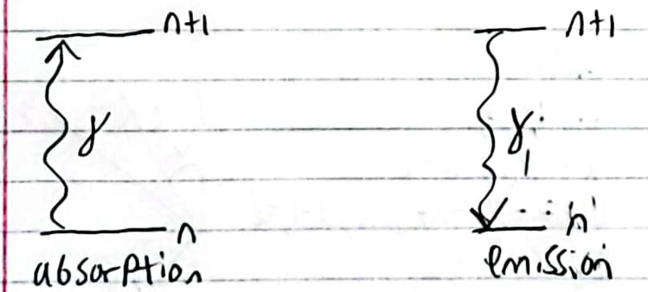


$$E_n = \frac{h^2}{8mL^2}\cdot n^2$$

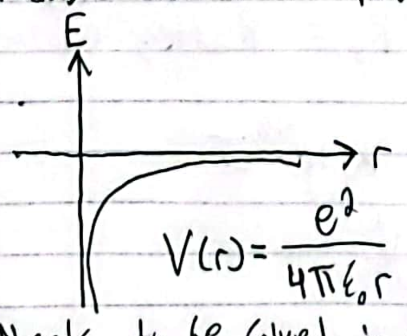
Box Model

- Eigenvalues are quantized in steps of 1

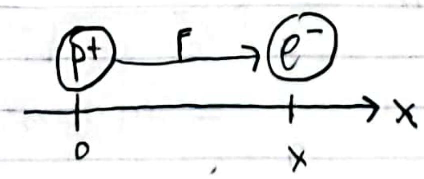
ex. Spin ½ has $m_s = -\frac{1}{2}\downarrow$ and $m_s = +\frac{1}{2}\uparrow$ oriantation



Today: One more potential V(x...): the H atom

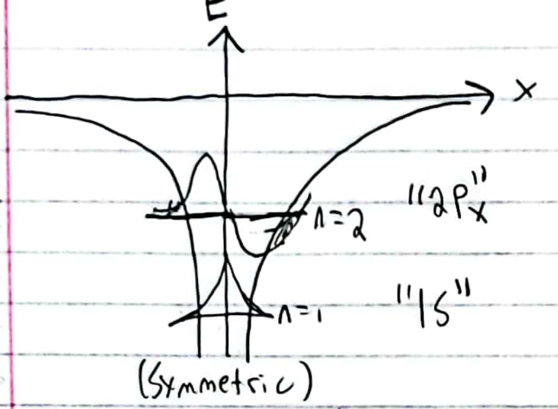


$$V(r) = \frac{e^2}{4\pi\varepsilon_0 r}$$



1-D picture $(r = |x|)$
$r > 0$

Needs to be solved in 4-D if we include the spin angle φ, but what might it look like projected onto just one dimension? x-axis



In 3-D $\mathcal{H}\psi_n = E_n \psi_n \Rightarrow$

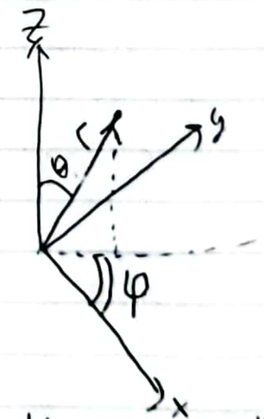
$$\left\{-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)-\frac{e^2}{4\pi\varepsilon_0 r}\right\}\psi = E_{n,l,m_l,m_s}\cdot\psi_{n,l,m_s,m_l}$$

(kinetic energy) (potential E)

The number of "quantum numbers" needed to index the solutions of a differential equation is equal to the number of variables

ex H atom has 4 quantum #s

Cartesian	Polar	Q.N.
x	r	$n = 1, 2, 3, \ldots$
y	θ	$l = 0, 1, \ldots, n-1$
z	φ	$m_l = -l, -l+1, \ldots, l-1, +l$
ψ_s	ψ_s	$m_s = -\frac{1}{2}, +\frac{1}{2}$



$$E = E_n = -\frac{R_y}{n^2}$$

$$R_y = \frac{m_e e^4}{8\varepsilon_0^2 h^2}$$ "R_y" = "Rydberg constant"

$$= 2.179673611035 \times 10^{-18}\ \text{J}$$

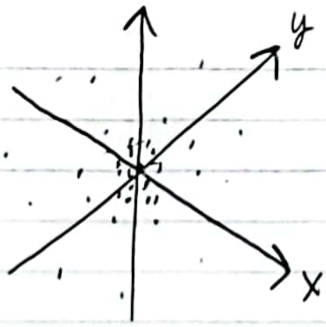
Wave functions:

$$\psi_{n=1, l=0, m_l=0} \sim e^{-r/a_0}$$

$$a_0 = \frac{4\pi\varepsilon_0 \hbar^2}{m_e e^2}$$

$\text{Prob.} \sim |\psi|^2 \sim e^{-2r/a_0}$

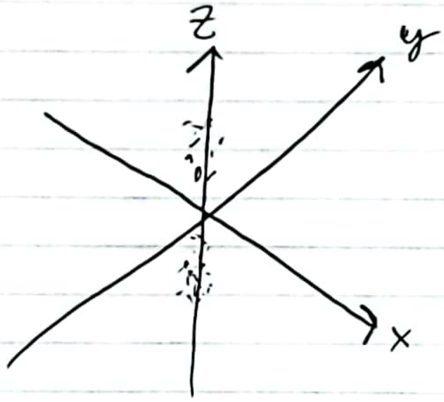
"dot plot"



Spherical Probability

"decaying sphere" probability
(less dense further out you go)

ex: $\psi_{2,1,0} \sim r e^{-r/2a_0} \cos\theta \quad \therefore \; P \sim |\psi|^2 \sim r^2 e^{-r/a_0} \cos^2\theta$



= $2p_z$ orbital